# Grading Rubic and Comments for Midterm

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## Question 1

(a) (+5)$u(r,v) = r^{N-1}v - b(r) - (N-1)\int_0^r b(x)\,dx$ where

- $r^{N-1}$ is probability of winning **(+1)**
- $v$ is the value of object won if he wins **(+1)**
- $b(r)$ is own bid **(+1)**
- $(N-1)\int_0^r b(x)\,dx$ is the expected sum of all the bids: number of bidders times the expected payment due to one bidder **(+2)**

(b) (+10) Derivative of $u(r,v)$ wrt $r$ is: **(+3)**

$$\frac{\partial u(r,v)}{\partial r} = (N-1)\,r^{N-2}v - b'(r) - (N-1)\,b(r)$$

maximized at $r = v$ **(+3)**

$$(N-1)\,v^{N-1} - b'(v) - (N-1)\,b(v) = 0.$$

Rearranging and caluculating (showing some steps **+2**),

$$b'(v) = (N-1)\left(v^{N-1} - b(v)\right)$$

Because assumption is $v^{N-1} > b(v)$, $b'(v) > 0$. **(+2)**

(c) (+10) Revenue is the **same**. **(+2)** because

- Probability assignment functions are the **same** (+**4**).
- $\bar{c}(0) = 0$ or $u(0) = 0$ or value 0 guy is **indifferent** (+**4**).
- If you show some ICDSM proof/argument, **+2**

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## Question 2

(a) (+10) A bidder's expected payoff from bidding $b(r)$ where $r \geq \rho$ when his value is $v \geq \rho$ is $u(r,v) = r^{N-1}(v - b(r))$ **(+1)**

Take derivative of $u(r,v)$ wrt $r$, $\frac{\partial u(r,v)}{\partial r} = (N-1)\, r^{N-2}(v - b(r)) - r^{N-1} b'(r)$ **(+1)**

Set it equal to 0 when $r = v$, $\frac{\partial u(r,v)}{\partial r}|_{r=v} = (N-1)\, v^{N-2}(v - b(v)) - v^{N-1} b'(v) = 0$ **(+1)**

Write it into differential equation form (but with steps), $\frac{d}{dv}\left(v^{N-1} b(v)\right) = (N-1)\, v^{N-1}$ **(+2)**

Some argument about $b(\rho) = \rho$ **(+2)**

Integrate, $v^{N-1} b(v) - \rho^{N-1} b(\rho) = \int_\rho^v (N-1)\, x^{N-1} dx = \frac{N-1}{N}\left(v^N - \rho^N\right)$ **(+1)**

Use $b(\rho) = \rho$, to get $b(v) = \frac{N-1}{N} v + \frac{1}{N}\rho\left(\frac{\rho}{v}\right)^{N-1}$. **(+2)**

(b) Revenue is **(+2)**

$$R = N \int_\rho^1 b(x)\, x^{N-1} dx$$

Plugging in $b$ and solve **(+2)**, and the result is **(+1)**

$$R = \frac{N-1}{N+1}\left(1 - \rho^{N+1}\right) + \rho^N (1 - \rho)$$

(c) A consultant maximizes $R$ **(+3)**

FOC is **(+2)**

$$-(N-1)\,\rho^N + N\rho^{N-1} - (N+1)\,\rho^N = 0$$

Some correct steps **(+3)** include

$$\begin{aligned} \rho^{N-1}(2N\rho - N) &= 0 \\ 2N\rho^N &= N\rho^{N-1} \\ 2N\rho &= N \end{aligned}$$

And optimal reserve price is $\rho^* = 1/2$ **(+2)**

## Question 3

(a) $u(r,v) = b(r)\, r^{N-1}(v - b(r))$ **(+1)**

Substitute, $u(r,v) = \alpha r^N (v - \alpha r)$ **(+1)**

Solve, $0 = \frac{\partial u}{\partial r} = N r^{N-1}(v - \alpha r) - r^N \alpha = Nv - N\alpha r - \alpha r = 0$ **(+2)**

Result is $b(v) = \frac{N}{N+1} v$ **(+1)**

(b) $\frac{N-1}{N} v < \frac{N}{N+1} v$, i.e. FPA less than this auction. **(+2)**

Reason: bidder is more aggresive in hidden reserve price auction because there is like another bidder. **(+3)**

(c) Show equivalence (can't write obvious: **+1**)

**Strictly prefer** an additional bidder, because the first N bidders bid the same (as shown), but for the N+1 (**+2**):

In FPA: it's $N/\left(N+1\right)\cdot v$, but (**+1**)

In hidden reserve price, it's $v$ (**+1**)

(d) Some points:

- FPA with optimal reserve price is revenue-maximizing
- Hidden reserve price auction deviates from the revenue-maximizing probability assignment function

## Question 4

(a) $p_i\left(v_1,\cdots,v_N\right)$(**+1**) $= 1$ if $v_i > v_j \quad \forall j \neq i$ (**+2**), and $= 0$ otherwise (**+2**)

(b) By ICDSM ii), $\bar{c}_i\left(v_i\right) = \bar{c}_i\left(0\right) + \bar{p}_i\left(v_i\right)v_i - \int_0^{v_i}\bar{p}_i\left(x\right)dx$ (**+5**)

(IR: $\bar{c}\left(0\right) \leq 0$)

(c) Any cost function that satisfies $\bar{c}\left(0\right) = 0$ and part (b) implies revenue-max subject to Eff and IR. One way is to set (**+5**)

$$c_i\left(v_i,\cdots,v_N\right) = p_i\left(v_1,\cdots,v_N\right)v_i - \int_0^{v_i} p_i\left(x,v_i\right)dx$$

Because it is possible of asymmetric bidders, SPA cost function is an example, but other auctions' cost functions are subject to distribution problems.

(d) SPA is **efficient**, **IR**, **IC** with $\mathbf{\bar{c}\left(0\right) = 0}$. (Notice the correspondence **+2**, argue each holds **+2**)