201 Notes: Intellectual Property

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This note expands and clarifies Glen's IP notes and solves the last two problems of PS5 along the way.

Supply of innovations: $S(\pi)$. Demand for the product is $Q(p)$ and supply of product is constant marginal cost, 0. We normalize p to be the **the fraction of the monopoly optimal price charged** and $Q(0)=1$. Then $Q(p)$ is the **fraction of the total efficient market served**. Profit

$$\pi(p) = pQ(p) \Rightarrow \frac{\partial \pi(p)}{\partial p} = Q(p) + pQ'(p)$$

and consumer surplus per product is

$$CS(p) = \int_p^\infty Q(\tilde{p})\,d\tilde{p} \Rightarrow \frac{\partial CS(p)}{\partial p} = -Q(p)$$

Then the social welfare is

$$\begin{aligned} SW(p) &= \text{consumer surplus} + \text{producer surplus} \\ &= S(\pi(p))\,CS(p) + \int_0^{\pi(p)} S(\pi)\,d\pi. \end{aligned}$$

To choose the socially efficient p, we look at FOC with respect to p,

$$\begin{aligned} \frac{\partial S(\pi(p^*))}{\partial \pi(p^*)}\frac{\partial \pi(p^*)}{\partial p}CS(p^*) + S(\pi(p^*))\frac{\partial CS(p^*)}{\partial p} + S(\pi(p^*))\frac{\partial \pi(p^*)}{\partial p} &= 0 \\ \frac{\partial S(\pi(p^*))}{\partial \pi(p^*)}\frac{1}{S(\pi(p^*))}\frac{\partial \pi(p^*)}{\partial p^*}CS(p^*) + \frac{\partial CS(p^*)}{\partial p} + \frac{\partial \pi(p^*)}{\partial p} &= 0 \\ \varepsilon_S(\pi(p^*))\frac{\partial \pi(p^*)}{\partial p^*}CS(p^*) - Q(p^*) + \frac{\partial \pi(p^*)}{\partial p} &= 0 \end{aligned}$$

And rearrange, we get

$$\varepsilon_S \frac{CS}{\pi} = \frac{DWL'}{\pi'}$$

where $DWL(p) = CS(0) - CS(p) - \pi(p)$.

Exercise 1 (LA2c)**.** We equate social welfare to consumer welfare, so FOC becomes

$$\begin{aligned} \varepsilon_S(\pi(p^*))\left[Q(p) + pQ'(p)\right]\frac{CS(p^*)}{\pi(p^*)} - Q(p^*) &= 0 \\ \varepsilon_S(\pi(p^*))\left[1 + pQ'(p)/Q(p^*)\right]\frac{CS(p^*)}{\pi(p^*)} &= 1 \\ \varepsilon_S(\pi(p^*))\frac{CS(p^*)}{\pi(p^*)} &= \frac{1}{1-\varepsilon_Q(p^*)} \end{aligned}$$

*All errors are solely mine. For questions, please email hanzhe@uchicago.edu.

Since $DWL + CS + \pi = K$,

$$DWL' = -\pi' - CS'$$

and

$$DWL'/\pi' = -1 - \frac{CS'}{\pi'} = -1 + \frac{Q}{Q + pQ'} = -\frac{pQ'}{Q + pQ'} = \frac{\varepsilon_Q}{1 - \varepsilon_Q} < \frac{1}{1 - \varepsilon_Q}$$

Therefore, the optimal price is lower in this case, the level of protection is lower (which is intuitive, as we place no weight on producer's welfare).

Exercise 2 (LA2b)**.** Since everything is put in fractional terms, holding elasticity of supply constant, everything is only shifted by fraction, although the optimal price is going to change, the optimal *fraction* of price will not be altered, especially with linear demand $Q(p) = a - bp$, as we demonstrate below.

$$\begin{aligned} CS(p) &= \frac{1}{2}\left(\frac{a}{b} - p\right)(a - bp) \\ \pi(p) &= (a - bp)\,p \\ DWL(p) &= \frac{1}{2}p\,(a - (a - bp)) = \frac{1}{2}p^2 b \end{aligned}$$

so

$$\begin{aligned} \pi'(p) &= a - 2bp \\ DWL'(p) &= bp \end{aligned}$$

However, we normalized $Q(0) = 1$, so $a = 1$, and $a - 2bp = 0$ for monopoly price $p = 1$, so $b = 0.5$. And we see that p^* that satisfies the equilibrium condition

$$\varepsilon \frac{CS(p^*)}{\pi(p^*)} = \frac{DWL'(p^*)}{\pi'(p^*)}$$

does not change with respect to change in supply or demand curve.