Evolutionary Justifications for Overconfidence

Kim Gannon Hanzhe Zhang

Michigan State University

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Overconfidence

- A person is overconfident when he or she overestimates his or her innate ability.
- There is overwhelming empirical evidence of confidence. It is probably the most well-established bias in psychology and economics: Oskamp (1965), Kidd (1970), Svenson (1981), Cooper et al. (1988), Bondt et al. (1989), Russo and Schoemaker (1992), Babcock and Loewenstein (1997), Guthrie et al. (2001).
- Perhaps overconfidence is an evolutionarily desirable trait...

Main Results

- This paper provides evolutionary justifications for overconfidence.
- We show in a standard two-player resource-fighting game, when players have heterogeneous and possibly wrong confidence levels about their chance of winning the game, overconfident players dominate in equilibrium.
  - Players may always know, or never know their opponent's confidence level.
  - The dominance of overconfident players always holds, regardless of the equilibrium players play.
- We do not need to assume any bias or constraint in Nature as in majority of previous work.

Literature: Survival of Biased Beliefs

- Optimal biased beliefs in strategic settings (indirect evolutionary approach): Güth and Yaari (1992), Heifetz et al. (2007a), Heifetz et al. (2007b), Dekel et al. (2007), Johnson and Fowler (2011).
- Optimal biased beliefs in incomplete asset markets (market selection hypothesis): Blume and Easley (1992), Sandroni (2000), Mailath and Sandroni (2003), Blume and Easley (2009b), Blume and Easley (2009a), Blume and Easley (2010), Beker and Chattopadhyay (2010), Coury and Sciubba (2012), Condie and Phillips (2016).
- Optimal biased beliefs of a single decision maker with other bias: Zhang (2013), Herold and Netzer (2015), Benabou and Tirole (2002), Compte and Postlewaite (2004), Zabojnik (2004), Benoit and Dubra (2011), Harris and Hahn (2011).

Motivating Example

- i and j play the following game. Their **payoffs** are

$i \setminus j$	Fight	Not Fight
Fight	$1_i \cdot 6 + 1_j \cdot 3 - 2, 1_j \cdot 6 + 1_i \cdot 3 - 2$	$6, 3$
Not Fight	$3, 6$	$3, 3$

  where 1_i, 1_j denote that i or j wins the game.
- In reality, i and j have the same chance of winning, $\Pr(1_i) = \Pr(1_j) = 0.5$.

Two Rational (Correct Belief) Players

- A type $\theta_i = 0.5$ player versus another type $\theta_j = 0.5$ player, their **perceived utilities** are the same as their payoffs.

$i \setminus j$	Fight	Not Fight
Fight	$\frac{1}{2}6 + \frac{1}{2}3 - 2 = 2.5, \frac{1}{2}6 + \frac{1}{2}3 - 2 = 2.5$	$6, 3$
Not Fight	$3, 6$	$3, 3$

- Three equilibria
  1. (Fight, Not Fight): $(6, 3)$.
  2. (Not Fight, Fight): $(3, 6)$.
  3. ($\frac{6}{7}\circ$Fight+$\frac{1}{7}\circ$Not Fight, $\frac{6}{7}\circ$Fight+$\frac{1}{7}\circ$Not Fight): $(3, 3)$.
- The two players on average get $x \in \{3, 4.5\}$.

An Overconfident Player

- A overconfident type $\theta_i = 0.8$ player **perceives** his utility to be

$i \setminus j$	Fight	Not Fight
Fight	$(0.8)(6) + (0.2)(3) - 2 = 3.4$	6
Not Fight	3	3

- Therefore, it is a **strictly dominant** strategy to play Fight.

Overconfident Player versus Rational Player

- An overconfident type $\theta_i = 0.8$ player and a rational type $\theta_j = 0.5$ player play the following game as they each observe the other player's confidence type,

$i \setminus j$	Fight	Not Fight
Fight	$3.4, 2.5$	$6, 3$
Not Fight	$3, 3$	$3, 3$

- The unique Nash equilibrium is ($\theta_i = 0.8$ Fight, $\theta_j = 0.5$ Not Fight),
- Equilibrium utilities and payoffs are 6 for the overconfident and 3 for the rational.

Two Overconfident Players

- Two overconfident types $\theta_i = 0.8$ and $\theta_j = 0.8$ players play their perceived game,

$i \setminus j$	Fight	Not Fight
Fight	$3.4, 3.4$	$6, 3$
Not Fight	$3, 3$	$3, 3$

- Both perceive a dominant strategy of Fight. The unique Nash equilibrium is (Fight, Fight).
- Both get an expected utility of 3.4 but an actual expected payoff of 2.5: the winner gets $4 = 6 - 2$ and the loser gets $1 = 3 - 2$.

Population Game

- In summary,
  - Two rational players: on average each gets $x \in [3, 4.5]$.
  - Overconfident player versus rational player: overconfident player gets 6 and rational player gets 3.
  - Two overconfident players: both Fight and each gets a payoff of 2.5.
- Population game
  - A continuum of players.
  - Players randomly pairwise match to play.
  - How many offspring (fitness) they have depend on the payoff.
  - In equilibrium, proportion p^* are overconfident.
  - Overconfident and rational players' payoffs equate.

Equilibrium Distribution of Confidence

- Proportion p^* are overconfident.
- Type 0.8 players' average payoff is

$$\pi^*_{0.8} = p^*(2.5) + (1 - p^*)(6)$$

- Type 0.5 players' average payoff is

$$\pi^*_{0.5} = p^*(3) + (1 - p^*)x$$

  where $x \in [3, 4.5]$.
- $\pi^*_{0.8} = \pi^*_{0.5}$ implies

$$\begin{aligned} 2.5p^* + (1 - p^*)6 &= 3p^* + (1 - p^*)x \\ (1 - p^*)(6 - x) &= 0.5p^* \\ \frac{1 - p^*}{p^*} &= \frac{0.5}{6 - x}. \end{aligned}$$

  Since $3 \leq x \leq 4.5$, $3/4 \leq p^* \leq 6/7$.
- 75% to 87% of players are descendants of overconfident players!

General Model

$i \setminus j$	Fight	Not Fight
Fight	$1_i \cdot R + 1_j \cdot r - c, 1_j R + 1_i r - c$	R, r
Not Fight	r, R	r, r

where

- $r < R$: It is worth to play Fight.
- $\frac{1}{2}(R - r) < c$: The rational player does not have a dominant strategy to Fight.
- $c < R - r$: It is a strictly dominant strategy for a type $\theta = 1$ player to Fight.

Population Game

- A continuum of players of confidence types $\theta \in [0, 1]$ is in the population.
- They randomly pairwise match to fight for the resource, observing each other's confidence type.
- A type θ_i player and a type θ_j player play the perceived game

$i \setminus j$	Fight	Not Fight
Fight	$\theta_i R + \theta_i r - c, \theta_j R + \theta_j r - c$	R, r
Not Fight	r, R	r, r

- Their payoffs are determined by

$i \setminus j$	Fight	Not Fight
Fight	$\frac{1}{2}R + \frac{1}{2}r - c, \frac{1}{2}R + \frac{1}{2}r - c$	R, r
Not Fight	r, R	r, r

Equilibrium when Confidence is Always Observed

Definition

An equilibrium consists of an equilibrium distribution of types represented by a CDF F^* and PDF f^* on $[0,1]$ and an equilibrium strategy $\sigma^*_\theta : [0,1] \to [0,1]$ such that

1. For each $\theta_i, \theta_j \in [0,1]$, $(\sigma^*_{\theta_i}(\theta_j), \sigma^*_{\theta_j}(\theta_i))$ is a Nash equilibrium in the perceived game played between type θ_i and θ_j players.
2. π_θ equalizes across all θ, where

$$\begin{aligned}\pi_{\theta_i} &= \int_0^1 [\sigma^*_{\theta_i}(\theta_j)\sigma^*_{\theta_j}(\theta_i)(\frac{1}{2}R + \frac{1}{2}r - c) \\ &\quad + \sigma^*_{\theta_i}(\theta_j)(1 - \sigma^*_{\theta_j}(\theta_i))R + (1 - \sigma^*_{\theta_i}(\theta_j))r] f^*(\theta_j) d\theta_j.\end{aligned}$$

Critical Type θ^*

- There is a critical confidence type $\theta^* = c/(R - r)$.
- For any type $\theta \geq \theta^*$ player, Fight is a dominant strategy.
- For any type $\theta < \theta^*$ player, Fight is not a dominant strategy.
- Since $c > (R - r)/2$, $\theta^* > 1/2$.
- Type $\theta \geq \theta^*$ players are **sup-critical** (certainly overconfident) and type $\theta < \theta^*$ players are called **sub-critical** (possibly overconfident).

Pairwise Games

- $\theta_i < \theta^*$ versus $\theta_j < \theta^*$: three equilibria (Fight, Not Fight), (Not Fight, Fight), (mixed) $\Rightarrow (R, r)$, (r, R), (r, r). The average payoff is $x \in [r, (R+r)/2]$.
- $\theta_i \geq \theta^*$ versus $\theta_j < \theta^*$: unique equilibrium (Fight, Not Fight) $\Rightarrow$ (R, r).
- $\theta_i \geq \theta^*$ versus $\theta_j < \theta^*$: unique equilibrium (Fight, Fight) $\Rightarrow$ $((R+r)/2 - c, (R+r)/2 - c)$.

Equilibrium Distribution

- Suppose proportion p^* are sup-critical and $1 - p^*$ are sub-critical.
- Average fitness of a sup-critical player:

$$\pi_{\theta \geq \theta^*} = p^*[(R + r)/2 - c] + (1 - p^*)R.$$

- Average fitness of a sub-critical player:

$$\pi_{\theta < \theta^*} = p^* r + (1 - p^*)x$$

  where $x \in [r, (R + r)/2]$.
- $\pi_{\theta \geq \theta^*} = \pi_{\theta < \theta^*}$ yields

$$p^* = \frac{R - x}{\frac{R+r}{2} + c - x}$$

Equilibrium Distribution

- Since $c < R - r$,

$$p > \frac{R - x}{R + \frac{1}{2}(R - r) - x} = 1 - \frac{\frac{1}{2}(R - r)}{R + \frac{1}{2}(R - r) - x}.$$

- Since $x \leq (R + r)/2$,

$$p^* > 1 - \frac{\frac{1}{2}(R - r)}{R + \frac{1}{2}(R - r) - \frac{1}{2}(R + r)} = \frac{1}{2}.$$

Proposition

In equilibrium, at least $1/2$ *of agents have confidence level above* $\theta^* = c/(R - r) > 1/2$.

Confidence is Never Observed

Definition

An equilibrium consists of an equilibrium distribution of types represented by a CDF F^* and PDF f^* on $[0,1]$ and an equilibrium strategy $\sigma^*_\theta : \emptyset \to [0,1]$ such that

1. For each $\theta_i \in [0,1]$,

$$\begin{aligned}\sigma^*_{\theta_i} \quad &\in \quad \arg\max_{\sigma\in[0,1]} \int_0^1 \{\sigma[\sigma^*_{\theta_j}(\theta_i R + (1-\theta_i)r - c) \\ &\quad + (1-\sigma^*_{\theta_j}(\theta_i))R] + (1-\sigma)r\} f^*(\theta_j)d\theta_j.\end{aligned}$$

2. π_θ equalizes across all $\theta \in [0,1]$, where

$$\begin{aligned}\pi_{\theta_i} \quad &= \quad \int_0^1 [\sigma^*_{\theta_i}\sigma^*_{\theta_j}(\frac{1}{2}R + \frac{1}{2}r - c) \\ &\quad + \sigma^*_{\theta_i}(1-\sigma^*_{\theta_j})R + (1-\sigma^*_{\theta_i}(\theta_j))r] f^*(\theta_j)d\theta_j.\end{aligned}$$

Critical Type θ^*

- There is a critical type θ^* who is indifferent between Fight and Not Fight.
- Any type $\theta > \theta^*$ player strictly prefers Fight and any type $\theta < \theta^*$ player strictly prefers Not Fight.
- Let p^* denote the proportion of type $\theta > \theta^*$ players, Δ^* proportion of type θ^* players, and $1 - \Delta^* - p^*$ proportion of type $\theta < \theta^*$ players.

Equilibrium Distribution of Confidence

- When a type θ player chooses Fight, his expected utility is

$$u_\theta = (p^* + \Delta^*\sigma^*)[\theta R + (1-\theta)r - c] + (1 - p^* - \Delta^*\sigma^*)R.$$

- It must hold for the critical type θ^*, $u_{\theta^*} = r$.

$$(p^* + \Delta^*\sigma^*)[\theta^* R + (1-\theta^*)r - c] + (1 - p^* - \Delta^*\sigma^*)R = r.$$

- Second, the fitnesses of all confidence types must equal,

$$(p^* + \Delta^*\sigma^*)[\frac{1}{2}R + (1-\frac{1}{2})r - c] + (1 - p^* - \Delta^*\sigma^*)R = r.$$

- From the two equations above, $\theta^* = 1/2$.
- And,

$$p^* + \Delta^*\sigma^* = \frac{R - r}{\frac{1}{2}(R-r) + c}.$$

  Since $c > R - r$, $p^* + \Delta^*\sigma^* > 2/3$. If there is no atom at $\theta^* = 1/2$, at least $2/3$ of the population are overconfident.

Conclusion

- Provides evolutionary game-theoretic justification for overconfident belief.
- In a class of resource-fighting games, under different settings of observability of players' beliefs, more players with overconfident beliefs survive.
- Remark 1: We only consider players of the same ability.
- Remark 2: We try to be agnostic about equilibrium selection and additional restrictions on evolutionary stability. Although we have possible multiple equilibria and a range of stable distribution of confidence levels, the main point that majority of players are overconfident in equilibrium holds with very little restriction.

THANK YOU!

References I

Babcock, Linda and George Loewenstein, "Explaining Bargaining Impasse: The Role of Self-Serving Biases," *Journal of Economic Perspectives*, Winter 1997, *11*, 190–126.

Beker, Pablo and Subir Chattopadhyay, "Consumption Dynamics in General Equilibrium: A Characterisation when Markets are Incomplete," *Journal of Economic Theory*, 2010, *145* (6), 2133–2185.

Benabou, Roland and Jean Tirole, "Self-Confidence and Personal Motivation," *Quarterly Journal of Economics*, 2002, *117* (3), 871–915.

Benoit, Jean-Pierre and Juan Dubra, "Apparent Overconfidence," *Econometrica*, September 2011, *79* (5), 1591–1625.

Blume, Lawrence and David Easley, "Evolution and Market Behavior," *Journal of Economic Theory*, 1992, *58* (1), 9–40.

References II

__ **and** __ , "The Market Organism: Long-Run Survival in Markets with Heterogeneous Traders," *Journal of Economic Dynamics and Control*, 2009, *33* (5), 1023–1035.

__ **and** __ , "Market Selection and Asset Pricing," in "Handbook of Financial Markets: Dynamics and Evolution," Elsevier, 2009, pp. 403–437.

__ **and** __ , "Heterogeneity, Selection, and Wealth Dynamics," *Annual Review of Economics*, 2010, *2* (1), 425–450.

Bondt, Kenneth A, Jeffrey A Frankel, Maria Sophia Aguirre, Reza Saidi, Paul R Krugman, and Maurice Obstfeld, "Forward Discount Bias: Is it an Exchange Risk Premium?," *Quarterly Journal of Economics*, 1989, *104* (9), 139–161.

Compte, Olivier and Andrew Postlewaite, "Confidence-Enhanced Performance," *American Economic Review*, 2004, *94* (5), 1536–1557.

References III

Condie, Scott S. and Kerk L. Phillips, "Can Irrational Investors Survive in the Long Run? The Role of Generational Type Transmission," *Economics Letters*, 2016, *139*, 40–42.

Cooper, Arnold C., Carolyn Y. Woo, and William C. Dunkelberg, "Entrepreneurs' Perceived Chance of Success," *Journal of Business Venturing*, 1988, *3* (3), 97–108.

Coury, Tarek and Emanuela Sciubba, "Belief Heterogeneity and Survival in Incomplete Markets," *Economic Theory*, January 2012, *49* (1), 37–58.

Dekel, Eddie, Jeffrey C. Ely, and Okan Yilankaya, "Evolution of Preferences," *Review of Economic Studies*, 2007, *74* (3), 685–704.

References IV

Güth, Werner and Menahem E. Yaari, "Explaining Reciprocal Behavior in Simple Strategic Games: An Evolutionary Approach," in U. Witt, ed., *Explaining Process and Change - Approaches to Evolutionary Economics*, Ann Arbor: University of Michigan Press, 1992.

Guthrie, Chris, Jeffrey J. Rachlinski, and Andrew J. Wistrich, "Inside the Judicial Mind: Heuristics and Biases," *Cornell Law Review*, 2001, *86* (4), 777–778.

Harris, Adam J L and Ulrike Hahn, "Unrealistic Optimism about Future Life Events: A Cautionary Note," *Psychological Review*, 2011, *118* (1), 135–154.

Heifetz, Aviad, Chris Shannon, and Yossi Spiegel, "The Dynamic Evolution of Preferences," *Economic Theory*, 2007, *32*, 251–286.

_, _, and _, "What to Maximize If You Must," *Journal of Economic Theory*, 2007, *133*, 31–57.

References V

Herold, Florian and Nick Netzer, "Second-Best Probability Weighting," March 2015. Working Paper.

Johnson, Dominic D. P. and James H. Fowler, "The Evolution of Overconfidence," *Nature*, September 2011, *477.*

Kidd, John B., "The Utilization of Subjective Probabilities in Production Planning," *Acta Psychologica*, 1970, *34* (C), 338–347.

Mailath, George J. and Alvaro Sandroni, "Market Selection and Asymmetric Information," *The Review of Economic Studies*, April 2003, *70* (2), 343–368.

Oskamp, Stuart, "Overconfidence in Case-Study Judgments," *Journal of Consulting Psychology*, 1965, *29* (3), 261–265.

Russo, J. Edward and Paul J. H. Schoemaker, "Managing Overconfidence," *Sloan Management Review*, 1992, *33* (2), 7–17.

References VI

Sandroni, Alvaro, "Do Markets Favor Agents Able to Make Accurate Predictions?," *Econometrica*, November 2000, *68* (6), 1303–1341.

Svenson, Ola, "Are We Less Risky and More Skillful Than Our Fellow Drivers?," *Acta Psychologica*, 1981, *47*, 143–148.

Zabojnik, Jan, "A Model of Rational Bias in Self-Assessments," *Economic Theory*, 2004, *23* (2), 259–282.

Zhang, Hanzhe, "Evolutionary Justifications for Non-Bayesian Beliefs," *Economics Letters*, November 2013, *121* (2), 198–201.