Evolutionary Justifications for Overconfidence

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Abstract

This paper provides evolutionary justifications for overconfidence. In each period, players are pairwise matched to fight for a resource and there is uncertainty about who wins the resource if they engage in the fight. Players have different confidence levels about their chance of winning although they actually have the same chance of winning in reality. Each player may know or may not know her opponent's confidence level. We characterize the evolutionarily stable equilibrium, represented by players' strategies and distribution of confidence levels. Furthermore, we characterize the evolutionary dynamics and the rate of convergence to the equilibrium. Under different informational environments, majority of players are overconfident, i.e. overestimate their chance of winning.

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1 Introduction

A person is overconfident when he or she overestimates his or her own ability. There is overwhelming evidence of population-wide overconfidence: Oskamp (1965), Kidd (1970), Svenson (1981), Cooper et al. (1988), Bondt et al. (1989), Russo and Schoemaker (1992), Babcock and Loewenstein (1997), Guthrie et al. (2001). The overwhelming evidence suggests that perhaps overconfidence is an evolutionarily desirable trait.

This paper provides evolutionary game-theoretic justifications for overconfidence. We show that in a resource-fighting game, with players of heterogeneous confidence levels about their own chance of winning the game, players with overconfident beliefs survive. The key improvement is that we do not need to assume any bias or constraint in Nature, in contrast to the majority of previous work. Rather, strategic interactions themselves result in the survival of biased beliefs and, in particular, that of overconfident beliefs.

The intuition for evolutionarily optimal overconfident beliefs is that rational players do not necessarily get the most payoff in strategic settings. In a game where players fight for resources, a rational perfect Bayesian player should not fight an overconfident player, but an overconfident player will fight a rational perfect Bayesian player who will back down as a result of the overconfident player's aggression. As a result, a rational player chooses not to fight with the overconfident player and the overconfident player wins when the two encounter.

Over the long run, there will be a distribution of players with heterogeneous beliefs about the state of the world. We show that, generically, over the long run, the majority of players will be overconfident. We show the robustness of the result both when a player knows his opponent's type and when a player does not know his opponent's type.

After a brief literature review, the paper is organized as follows. Section 2 provides a motivating example. Section 3 describes the general model. Section 4 solves the general model when confidence types are observed. Section 5 solves the general model when confidence types are not observed. Section 6 concludes.

1.1 Literature Review

1.1.1 Biased Beliefs of a Single Decision Maker

When a player departs from being a risk-neutral utility maximizer, having a biased belief may correct other inherent biases. Zhang (2013) shows that when a decision maker is risk-averse, the evolutionarily optimal belief is positively biased. Herold and Netzer (2015) show that non-Bayesian probability weighting is a way to correct for S-shaped value function that arises in prospect theory, and make the general point that non-Bayesian probability weighting is the correction to biases in non-linear utilities.[1] The common thread in all these works is that there is some constraint in Nature, and evolution works to find the second best

[1] The natural question why Bayesian and linear utilities do not evolve to survive and dominate other sub-optimal second-best probability weighting and strategies is addressed in Ely (2011). He gives analogies in Microsoft Office updates: the system is not completely overhauled to reach the first-best solution; rather, patches and what he calls "kludges" are added over time. Similarly, evolution works by adding patches rather than completely overhauling the system.

solution. One a priori bias results in an ex post bias in another dimension (e.g. risk aversion results in overconfident, S-shaped utility function results in non-linear probability weighting). Benabou and Tirole (2002) and Compte and Postlewaite (2004) also point out the desirable effects of self-perception for a single decision maker who exhibits other constraints.

Zabojnik (2004), Benoit and Dubra (2011), and Harris and Hahn (2011) try to rationalize observed overconfidence, but the empirical evidence for overconfidence is too overwhelming to be disregarded.

1.1.2 Biased Beliefs in Strategic Interactions

Güth and Yaari (1992) and Güth (1995) developed the so-called indirect evolutionary approach which we use in this paper: players choose actions to maximize their perceived utilities but their actual payoffs and fitness differ from their perceived utilities. Heifetz et al. (2007a) and Heifetz et al. (2007b) show in general that the rational players are not the ones who survive in an evolutionary process. Because the games are assumed to be dominance solvable and have a unique Nash equilibrium, only one type of player survives in the limit. Instead, in this paper, we deal with games with multiple equilibria and a distribution of players with different belief types will survive. Dekel et al. (2007) deal with two by two games potentially with multiple equilibria and characterize stable distributions of preferences. The game we consider in this paper does not have a stable distribution according to their definition of stability (Proposition 3 in Dekel et al. (2007)). For our purpose of showing the limit distribution of overconfidence, we do not need to restrict our attention to any particular equilibrium. The paper most closely related to the current paper is Johnson and Fowler (2011). However, their paper treats biased estimation of winning probability as a “mistake" and also does not discuss the cases where the players' types are not observed at all. Other papers also mention the advantage of being an overconfident type in Tullock contests: see Ludwig et al. (2011).

1.1.3 Market Selection Hypothesis

The market selection hypothesis literature investigates the survival of players with different beliefs in a competitive market, instead of pairwise strategies interactions in the indirect evolutionary games. Blume and Easley (1992) show that when players maximize their discounted log utilities of consumption, the players who have correct beliefs about the state of the world will dominate the asset market in the long run. Sandroni (2000) endogenizes the savings decision in Blume and Easley (1992) and shows that regardless of the players' utility functions, the players with correct beliefs will dominate. A crucial assumption is that markets are complete. Subsequent papers deal with incomplete markets, and find that players with non-Bayesian beliefs may survive: Mailath and Sandroni (2003), Blume and Easley (2009b), Blume and Easley (2009a), Blume and Easley (2010), Beker and Chattopadhyay (2010), Coury and Sciubba (2012), Condie and Phillips (2016).

2 Motivating Example

Let's use a motivating example with specific payoffs to illustrate the key insight of the paper. The rest of the paper generalizes the key insight in the same game with general payoffs.

Two players (e.g. two countries, two lions, or two firms) both want an indivisible resource (e.g. a land, a lioness, or a market). If they do not Fight for it, neither would get the resource and both get a payoff of $r = 3$. If one of them Fights, the one who Fights gets $R = 6$ and the one who does Not Fight gets $r = 3$. If both Fight, they engage in a costly encounter that costs $c = 2$ to each and the winner gets $R = 6$ and the loser gets $r = 3$. (The payoffs $R = 6$, $r = 3$, and $c = 2$ used in the motivating example are the smallest integers that satisfy the general game described in the next section.)

In an evolutionary context, they are naturally occurring games in the nature when two parties of the same specifies or two different specifies fight for resources such as food, land, and money. consider these payoffs as their fitness levels, the (expected) number of offspring. The game is summarized as follows, where 1_i, 1_j denote that i or j wins the resource.

$i \setminus j$	Fight	Not Fight
Fight	$1_i \cdot 6 + 1_j \cdot 3 - 2,\ 1_j \cdot 6 + 1_i \cdot 3 - 2$	$6, 3$
Not Fight	$3, 6$	$3, 3$

Everyone in the population actually has the same chance of winning, that is, $\Pr(i \text{ wins}) = \Pr(i \text{ loses}) = 0.5$. A player with the correct belief that he wins with probability 0.5 perceives his expected utility to be the same as the fitness payoff,

$i \setminus j$	Fight	Not Fight
Fight	$(0.5)(6) + (0.5)(3) - 2 = 2.5$	6
Not Fight	3	3

A player may not have the correct belief that he wins with probability of 0.5. A player with belief that he wins with probability $\theta_i \in [0, 1]$ perceives his expected utility to be

$i \setminus j$	Fight	Not Fight
Fight	$(\theta_i)(6) + (1 - \theta_i)(3) - 2 = 3\theta_i + 1$	6
Not Fight	3	3

Note that when the opponent Fights, a type $\theta_i > 2/3$ player's perceived expected utility when he Fights is greater than his utility when he does Not Fight. In particular, Fight is a strictly dominant strategy for a confidence type $\theta_i > 2/3$. In other words, a sufficiently overconfident player always Fights regardless of the situation.

2.1 Confidence Types are Observed

First, suppose that each player knows her opponent's confidence type. She chooses her action based on her own confidence type and her opponent's observed confidence type. Suppose the players are either rational with confidence levels of 0.5 or overconfident with confidence levels of 0.8.

2.1.1 Static Pairwise Game

(a) Rational Player versus Rational Player When two players who have correct beliefs about their chance of winning play the game, they are playing the actual game

$i \setminus j$	Fight	Not Fight
Fight	2.5, 2.5	$6, 3$
Not Fight	$3, 6$	$3, 3$

There are three equilibria: (F, NF), (NF, F), and ($\frac{6}{7}\circ$F$+\frac{1}{7}\circ$NF, $\frac{6}{7}\circ$F$+\frac{1}{7}\circ$NF). In the two pure-strategy equilibria, the one who fights gets 6 and the one who does not fight gets 3; in the mixed-strategy equilibrium, both players get an expected payoff of 3.

(b) Overconfident Player versus Rational Player Now suppose an overconfident player i with belief $\theta_i = 0.8$ and a rational player j with belief $\theta_j = 0.5$ play against each other and their beliefs are observable and common knowledge. The game they believe they are playing is then

$\theta_i = 0.8 \setminus \theta_j = 0.5$	Fight	Not Fight
Fight	3.4, 2.5	6
Not Fight	3	3

The overconfident player i has a dominant strategy of Fight. The rational player j on the other hand has an expected utility of 2.5 from fighting when i fights. When i fights, j's best response is Not Fight. As a result, the equilibrium is that i plays Fight and j plays Not Fight, and i the overconfident player gets 6 and j the Bayesian player gets 3.

(c) Overconfident Player versus Overconfident Player If two overconfident type $\theta_i = \theta_j = 0.8$ players encounter each other, their perceived expected utilities are

$i \setminus j$	Fight	Not Fight
Fight	3.4, 3.4	$6, 3$
Not Fight	$3, 6$	$3, 3$

whereas their true expected payoffs are

$i \setminus j$	Fight	Not Fight
Fight	2.5, 2.5	$6, 3$
Not Fight	$3, 6$	$3, 3$

They both play Fight and get an actual payoff of 2.5.

2.1.2 Evolutionarily Stable Distribution of Confidence

Consider a distribution of players with different beliefs. For example, start with half a population of rational type 0.5 players and half a population of overconfident type 0.8 players. They randomly match to play the game just described knowing the belief type of the opponent. Within a generation, players play the game repeatedly. Their average payoffs determine the growth rate of the population.

We focus on an evolutionarily stable equilibrium such that (1) two players meet and play one of the Nash equilibria of the perceived game, and (2) the average fitness of the two types of players is the same. Formally, an evolutionarily stable equilibrium specifies that (1) each type θ_i player's probability $\sigma_{\theta_i}(\theta_j)$ of Fight when he encounters a type θ_j player, and (2) proportion p^* of players are overconfident and proportion $(1-p^*)$ of players are rational, such that the expected payoffs are the same for the two types of players.

When two rational players match, each player's payoff is x between 3 and 6 depending on the equilibrium they play, but the sum of the payoffs is less than 9 for the two players, so each rational player has an equilibrium payoff smaller than 4.5 on average. When an overconfident player plays with a rational player, the overconfident player plays Fight and gets 6 and the rational player plays Not Fight and gets 3. When two overconfident players meet, they both play Fight. Ex post one gets 4 and one gets 1; ex ante two players' average payoff is 2.5.

Suppose in the evolutionarily stable distribution, a proportion p of players are overconfident and proportion $1-p$ are rational. Overconfident players have an average payoff of

$$(p)(2.5) + (1-p)(6).$$

Rational player have an average payoff of

$$(p)(3) + (1-p)x$$

where x is between 3 and 4.5. In the stable equilibrium, there is a proportion p^* of overconfident players. Overconfident and rational players have the same expected payoff:

$$(p^*)(2.5) + (1-p^*)(6) = (p^*)(3) + (1-p^*)x \Rightarrow p^* = \frac{6-x}{6.5-x} = 1 - \frac{0.5}{6.5-x}.$$

Since $3 \leq x \leq 4.5$,

$$1 - \frac{0.5}{6.5-4.5} = \frac{3}{4} \leq p^* \leq 1 - \frac{0.5}{6.5-3} = \frac{6}{7}.$$

That is, in an evolutionary stable equilibrium, between 75% and 85.7% of the players are overconfident (recall 75% of Harvard undergraduates believe that they are better their median Harvard peer Möbius et al. (2012) and 88% of drivers believe they have above -median driving skills in Svenson (1981)), and only between 14.3% and 25% of the players are rational and have correct beliefs.

2.1.3 Evolutionary Dynamics of Confidence

With our simple model, we can also investigate numerically how long it takes for the population to reach the stable distribution of overconfidence levels. Figure 2.1.3 illustrates the evolutionary dynamics when initially there is an equal proportion of rational and overconfidence players is 0.5. Then, for each succeeding generation, the proportion of overconfident players is calculated as the proportion of expected number of offspring to overconfident players over the entire population. The expected number of offspring is calculated based on the probability of encountering a player of each type and the payoff under each match:

$$p_t(p_{t-1}) = \frac{p_{t-1}[6(1-p_{t-1}) + 2.5p_{t-1}]}{p_{t-1}[6(1-p_{t-1}) + 2.5p_{t-1}] + (1-p_{t-1})[4.5(1-p_{t-1}) + 3p_{t-1}]}.$$

Here, the expected payoff to a rational player who plays a rational player is taken to be 4.5 based on the probability that the player plays one of the pure strategy equilibria or the mixed strategy equilibrium. Generally, the proportions tend to be equal within three decimal places of each other beginning around the fortieth generation. We take the proportion $p_{50} = p^*$, the limit of this sequence.

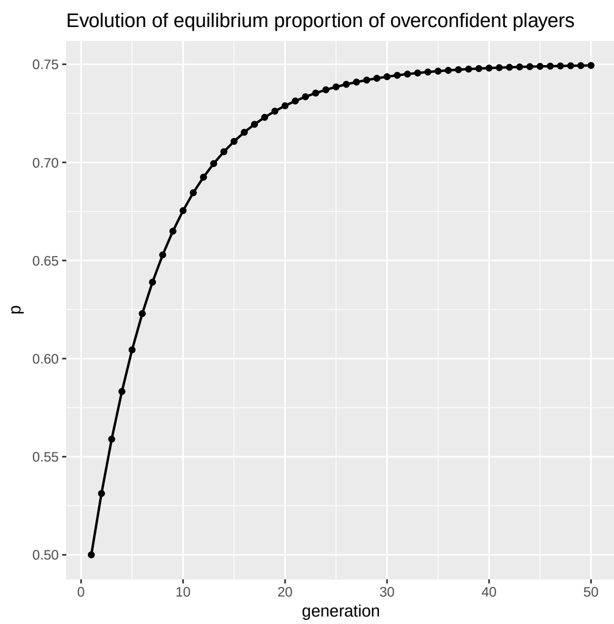

Figure 1: Evolutionary dynamics when $p_0 = 0.5$, $R = 6$, $r = 3$, and $c = 2$.
The figure illustrates 50 generations of the evolutionary dynamics of confidence when confidence types are always observed, the initial proportion of overconfident players is $p_0 = 0.5$, the winner's payoff is $R = 6$, the loser's payoff is $r = 3$, and cost of fighting is $c = 2$. The proportion of overconfident players in the population approaches 0.75, the evolutionarily stable proportion, in approximately 30 generations.

Next, we calculate the rate of convergence of the sequence $\{x_t^*\}$ by fixed point iteration. This method is based on the linear order convergence of $\{p_t\}$ and assumes that the deviation $e_t = |p_t - p^*| \leq \lambda\rho$, $p^* \in [p_0 - \rho, p_0 + \rho]$ for $p_0 \in \{p_t\}$. Here, λ^t is the convergence rate. We then calculate λ by curve-fitting

$$\log(e_t) = t\log(\lambda) + \log(\rho)$$

and transforming the coefficient on t.

From this analysis, we estimate that the proportion of overconfident players in a population converges to $p^* = 0.749 \approx 0.750$, our true equilibrium distribution as calculated in the previous subsection. Our curve-fitting procedure gives a coefficient of -0.1443, yielding a growth rate of $\lambda^k = (e^{-0.1443})^k = 0.866^k$. The deviation between p_t and the limit p^* decreases by approximately 13.4% each generation.

2.2 Confidence Types are Not Observed

Second, suppose that a player cannot observe his opponent's confidence type but knows the distribution of confidence types in a period. Each player plays against a population of players with heterogeneous confidence levels. Each player chooses an action based on his own confidence level and the aggregate distribution of confidence levels and strategies. Suppose that there are just two types of agents as in the previous case: the overconfident type, whose believed probability of victory is 0.8, and the rational type, whose is 0.5.

2.2.1 Static Pairwise Game

The overconfident type, regardless of the situation, Fights, as Fighting is an overconfident player's dominant strategy. A rational player weighs the benefits and costs of Fighting. Given that proportion p_t of players is overconfident, a rational player's expected utility of Fighting is

$$\begin{aligned} & p_t[0.5(6)+0.5(3)-2]+(1-p_t)[\underbrace{\sigma(0.5,p_t,\text{Fight})}_{\equiv\sigma_t}(2.5)+(1-\sigma(0.5,p_t,\text{Fight}))(6)] \\ = \quad & [1-(1-p_t)(1-\sigma_t)](2.5)+(1-p_t)(1-\sigma_t)(6) \quad \text{vs} \quad 3 \text{ if he chooses Not Fight.} \end{aligned}$$

If $(1-p_t)(1-\sigma_t) = 1/7$, then the rational player is indifferent between Fighting and Not Fighting. Therefore, the probability of a rational player Fighting when the proportion of overconfident players is p_t is

$$\sigma_t \equiv \sigma(0.5, p_t, \text{Fight}) = 1 - \frac{1}{7(1-p_t)}.$$

This distribution is independent across time periods. Therefore, when each player cannot observe the other players' individual types but an aggregate distribution of other players' confidence types, there exist only trivial evolutionary dynamics. Nonetheless, extremely under-confident players will be driven out of the market.

2.2.2 Evolutionarily Stable Distribution of Confidence

As above, we focus on the evolutionary stable equilibrium. In equilibrium, a type θ player plays Fight with probability σ^*_θ. Proportion p^* of the population is overconfident, such that, in equilibrium, (1) σ^*_θ is best responding to the situation in which a player is playing with probability p^* an overconfident player who adopts strategy $\sigma^*_{0.8}$ and with probability $1-p^*$ a rational player who adopts strategy $\sigma^*_{0.5}$, and (2) the expected payoffs of the two types of players are the same.[2]

[2] $\emptyset$ in σ_θ indicates that the player receives no information about his opponent's type.

When an overconfident player plays Fight, his expected utility is

$$u_{0.8} = p[(\sigma_{0.8})(3.4) + (1 - \sigma_{0.8})(6)] + (1 - p)[(\sigma_{0.5})(3.4) + (1 - \sigma_{0.5})(6)] > 3$$

where 3 is the expected utility of Not Fight. Therefore, an overconfident player always plays Fight, and thus $\sigma^*_{0.8} = 1$. A rational player playing Fight gets an expected utility of

$$u_{0.5} = p(2.5) + (1 - p)[(\sigma_{0.5})(2.5) + (1 - \sigma_{0.5})(6)].$$

In order for a rational player to be indifferent between Fight and Not Fight,

$$[1 - (1 - p^*)(1 - \sigma^*_{0.5})](2.5) + (1 - p^*)(1 - \sigma^*_{0.5})(6) = 3 \Rightarrow (1 - p^*)(1 - \sigma^*_{0.5}) = 1/7.$$

$1 - p^* \geq 1/7$ and $1 - \sigma^*_{0.5} \geq 1/7$, so $p^* \leq 6/7$ and $\sigma^*_{0.5} \leq 6/7$. There could be proportion $p^* \in [0, 6/7]$ of overconfident players. Unless $p^* = 0$, the average confidence level, $(p^*)(0.8) + (1 - p^*)(0.5)$ strictly exceeds the rational level of 0.5, and the majority of players are weakly overconfident.

We will show that the claim that majority of players are weakly overconfident holds in general, even when there are under-confident players.

2.2.3 Evolutionary Dynamics of Confidence

Now let's consider the evolution of distribution of overconfident players, $p_t(p_{t-1})$. Consider random pairwise matching between all the players. The expected payoff of an overconfident player is

$$\begin{aligned}
& p_t(2.5) + (1 - p_t)[(\sigma_t)(2.5) + (1 - \sigma_t)(6)] \\
= \;& p_t(2.5) + (1 - p_t)\left[\left(1 - \frac{1}{7(1 - p_t)}\right)(2.5) + \frac{1}{7(1 - p_t)}(6)\right] \\
= \;& p_t(2.5) + (1 - p_t)\left[2.5 + \frac{3.5}{7(1 - p_t)}\right] = 3.
\end{aligned}$$

The expected payoff of a rational player is

$$\begin{aligned}
& p_t[(\sigma_t)(2.5) + (1 - \sigma_t)(3)] + (1 - p_t)[(\sigma_t^2)(2.5) \\
& +(\sigma_t)(1 - \sigma_t)(6) + (1 - \sigma_t)(\sigma_t)(2) + (1 - \sigma_t)^2(3)] \;=\; 3.
\end{aligned}$$

In this example, the proportion of overconfident players in a population p_t is constant at 3 does not depend on that from a previous period p_{t-1}. Therefore, if we assume that the payoff to each player is the number of offspring in the next generation, the distribution of overconfident players in a population does not change over time:

$$p_t = \frac{p_{t-1}(3)}{p_{t-1}(3) + (1 - p_{t-1})(3)} = \frac{p_{t-1}(3)}{3} = p_{t-1}.$$

Therefore, under the parameters of this game, a population's distribution of overconfident players will not change over time if opponents' confidence types are never known.

3 General Model

There is a continuum of players with heterogeneous beliefs $\theta \in [0,1]$ about their chance of winning. Let μ denote the measure of players with different beliefs. The fitness payoff is

$i \setminus j$	Fight	Not Fight
Fight	$1_i R + 1_j r - c$, $1_j R + 1_i r - c$	R, r
Not Fight	r, R	r, r

where $r < R$ and $\frac{1}{2}(R-r) < c < R - r$. $\frac{1}{2}(R-r) < c$ implies that a rational player would not fight if the opponent fights. $c < R - r$ implies that a player who believes he always wins would prefer to fight even if the opponent fights, i.e. a player of type $\theta = 1$ has a strictly dominant strategy of Fight. With probability q, a player observes his opponent's play.

4 Confidence Types are Observed

Suppose for now that opponents' confidence types are always observable. In this section, we solve for the pairwise static games, players' payoffs, and the equilibrium distribution, and the evolutionary dynamics.

4.1 Pairwise Static Games

Two players of confidence confidence types $\theta_i \in [0,1]$ and $\theta_j \in [0,1]$ play the following perceived game,

$i \setminus j$	Fight	Not Fight
Fight	$\theta_i R + (1-\theta_i)r - c$, $\theta_j R + (1-\theta_j)r - c$	R, r
Not Fight	r, R	r, r

Any type θ player who decides to Fight receives a minimum expected payoff of $\theta R + (1-\theta)r - c$ (in the case that the opposing player fights for sure). The alternative is Not Fight, and the type θ player gets r. Consequently, a player will definitely Fight if

$$\theta R + (1-\theta)r - c > r,$$

which rearranges to

$$\theta > \frac{c}{R-r} = \theta^*$$

where we call θ^* the *critical type*. Since $c > (R-r)/2$ by assumption, the critical type $\theta^* > 1/2$. Any player with type $\theta < \theta^*$ is called *under-critical* and any player with type $\theta \geq \theta^*$ is called *over-critical*. Since $\theta^* > 1/2$, an over-critical player is necessarily over-confident, a rational player is necessarily under-critical, but some under-critical players are also overconfident.

For any over-critical $\theta \geq \theta^*$ player, Fight is a dominant strategy. For any under-critical $\theta < \theta^*$ player, Fight is not a dominant strategy. In particular, for a rational $\theta = 1/2 < \theta^*$ player, Fight is never a dominant strategy.

(a) Under-critical Type $\theta_i < \theta^*$ Player versus Under-critical Type $\theta_j < \theta^*$ Player. Suppose that two under-critical players encounter each other. In particular, two rational $\theta = 1/2$ players encountering each other is a special case. There are three equilibria: two pure-strategy equilibria (Fight, Not Fight) and (Not Fight, Fight) and one mixed-strategy equilibrium $(\sigma^*_{\theta_i}(\theta_j),\ \sigma^*_{\theta_j}(\theta_i))$. In the two pure-strategy Nash equilibria, one player gets R and the other gets r, and the total payoff is $R + r$. On average, the two players on average get a payoff $(R + r)/2$. In the unique mixed-strategy Nash equilibrium, each person gets r in expectation. Therefore, each $\theta_i, \theta_j < \theta^*$ player gets a payoff $x \in [r, (R + r)/2]$.

(b) Overconfident $\theta_i \geq \theta^*$ Player versus Type $\theta_j < \theta^*$ Player. It is a dominant strategy to play Fight. Given that a type $\theta_i \geq \theta^*$ player plays Fight, the $\theta_j < \theta^*$ player plays Not Fight. The unique Nash equilibrium is (Fight, Not Fight), and the type $\theta_i > \theta$ player gets R and the type $\theta_j < \theta^*$ player gets r.

(c) Overconfident $\theta_i \geq \theta^*$ Player versus Overconfident $\theta_j \geq \theta^*$ Player. Two overconfident players both choose Fight and they end up with an expected payoff of $(R+r)/2-c$.

4.2 Evolutionarily Stable Distribution of Confidence

Definition 1. An equilibrium consists of a CDF F^* with associated PDF f^* on $[0, 1]$ representing the distribution of types, and a strategy $\sigma^*_\theta : [0, 1] \cup \emptyset \to [0, 1]$ such that

1. For each $\theta_i, \theta_j \in [0, 1]$,

$$\sigma^*_{\theta_i}(\theta_j) \in \arg\max_{\sigma\in[0,1]} \sigma[\sigma^*_{\theta_j}(\theta_i)(\theta_i R + (1 - \theta_i)r - c) + (1 - \sigma^*_{\theta_j}(\theta_i))R] + (1 - \sigma)r,$$

   or more loosely, to incorporate asymmetric strategies, $(\sigma^*_{\theta_i}(\theta_j), \sigma^*_{\theta_j}(\theta_i)) \in \mathcal{N}(\theta_i, \theta_j)$, where $\mathcal{N}(\theta_i, \theta_j)$ denotes the set of Nash equilibria in the perceived game played between types θ_i and θ_j players.

2. π_θ equalizes across all θ, where

$$\pi_{\theta_i} = \int_0^1 [\sigma^*_{\theta_i}(\theta_j)\sigma^*_{\theta_j}(\theta_i)(\frac{1}{2}R + \frac{1}{2}r - c) + \sigma^*_{\theta_i}(\theta_j)(1 - \sigma^*_{\theta_j}(\theta_i))R + (1 - \sigma^*_{\theta_i}(\theta_j))r]f^*(\theta_j)d\theta_j.$$

We can show that the stable proportion of overconfident players is always above 1/2. Suppose p^* is the proportion of players with types above θ^*. Then, an overconfident $\theta \geq \theta^*$ player gets a payoff of

$$\pi_{\theta>\theta^*} = p^*[(R + r)/2 - c] + (1 - p^*)R$$

and a type $\theta < \theta^*$ player gets a payoff of

$$\pi_{\theta\leq\theta^*} = p^*r + (1 - p^*)x$$

where $x \in [r, (R + r)/2]$. In equilibrium, the two equate,

$$p^*[(R + r)/2 - c] + (1 - p^*)R = p^*r + (1 - p^*)x$$

which simplifies to

$$p^* = \frac{R-x}{\frac{R+r}{2} + c - x}$$

Since $c < R - r$,

$$p^* > \frac{R-x}{\frac{R+r}{2} + R - r - x} = \frac{R-x}{R + \frac{1}{2}(R-r) - x} = 1 - \frac{\frac{1}{2}(R-r)}{R + \frac{1}{2}(R-r) - x}$$

Since $x \leq (R+r)/2$,

$$p^* > 1 - \frac{\frac{1}{2}(R-r)}{R + \frac{1}{2}(R-r) - x} \geq 1 - \frac{\frac{1}{2}(R-r)}{R + \frac{1}{2}(R-r) - \frac{1}{2}(R+r)} = \frac{1}{2}.$$

That is, in equilibrium, it always holds that more than half of the agents have a confidence level above $\theta^* > (R-r)/c > 0.5$. The exact distribution of confidence levels depends on the initial distribution one starts with, but we have shown that regardless of the initial distribution, at least half of the population have strictly positively biased beliefs.

We can also look at the change in the evolutionarily stable proportion of overconfident players. The stable proportion of overconfident players increases when the winning payoff R increases, when the losing payoff r decreases, and when the cost of fighting c decreases.

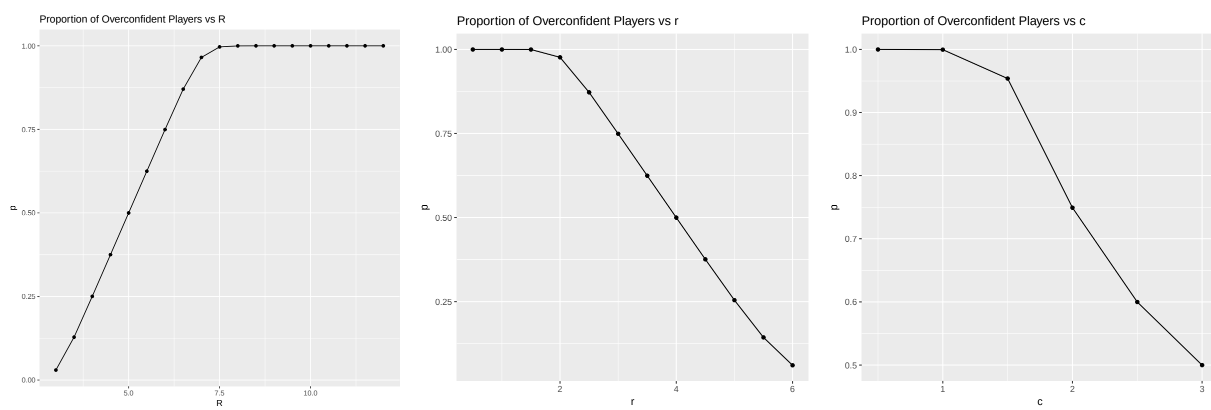

Figure 2: Evolutionarily stable proportion of overconfident players, when R, r, and c vary.

Our numerical analysis also allows variation on various parameters within the model. Figure 2 notes the differences in the evolutionarily stable proportion of overconfident players for different values of R holding constant the values of r and c. Indeed, we note that with a value R less than 5, the long run proportion of overconfident players lies beneath the initial proportion 0.5. At $R = 5$, the value of p remains constant at 0.5, and any value of R greater than 5 yields an increase in p. We note that around roughly $R = 7.5$, the proportion of overconfident players begins to approach 1 in the long run.

Figure 3 illustrates the change in the stable proportion of overconfident players for different values of winning payoff R, losing payoff r, cost c, initial proportion of overconfident players.

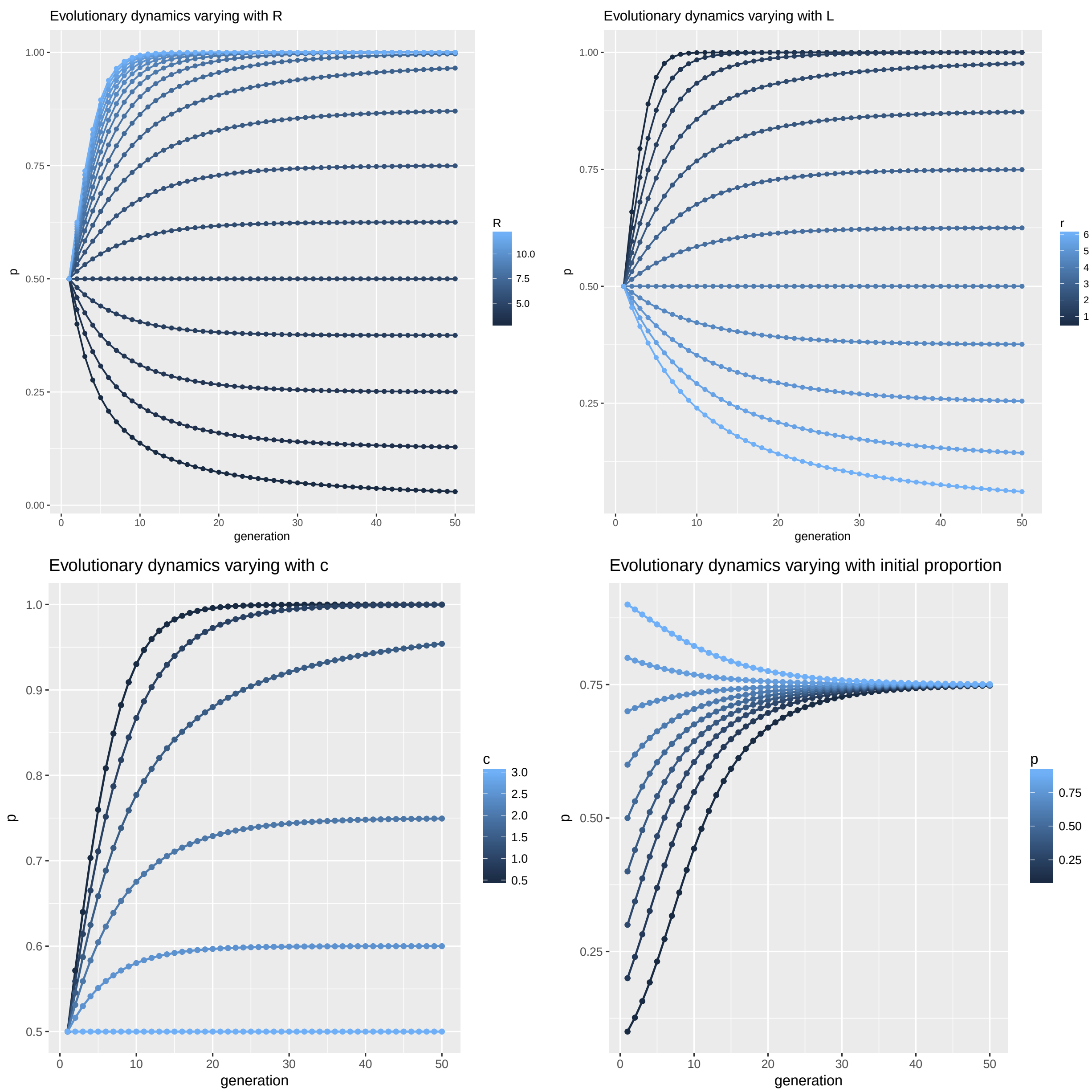

Figure 3: Evolutionary dynamics, when R, r, c and p_0 vary.

4.3 Evolutionary Dynamics

Figure 4 plots the number of time periods necessary for the population to reach within 0.01 of the evolutionarily stable distribution. Somewhat surprisingly, the number of generations to reach within 0.01 of the equilibrium proportion is not monotonic in R, r, c, p_0. The variations of convergence rates deserve attention from future research.

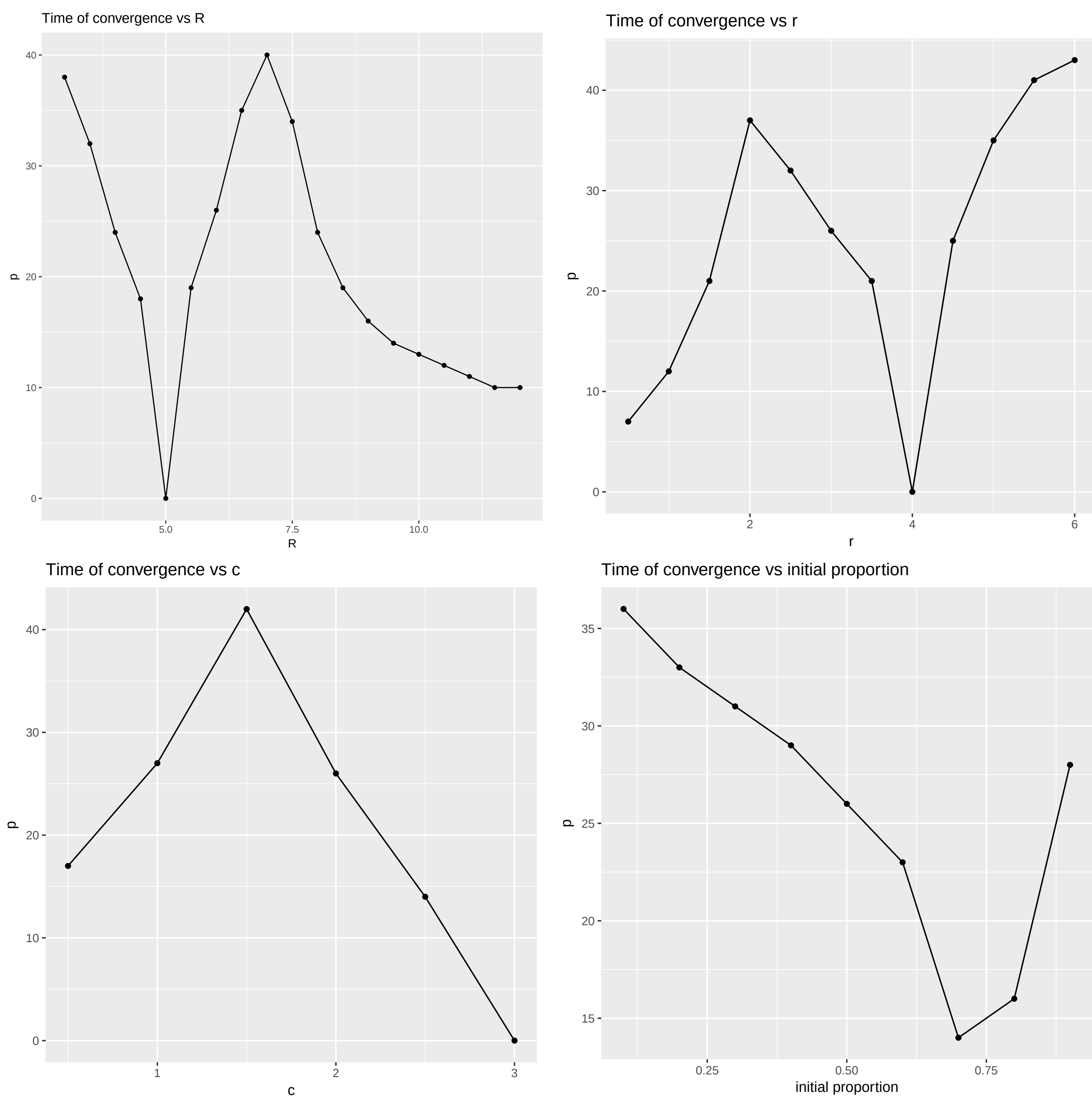

Figure 4: Number of generations to reach equilibrium, when R, r, c and p_0 vary.

5 Confidence Types are Not Observed

Suppose that players cannot observe their opponent's confidence type. We are interested in characterizing the equilibrium distribution of confidence levels and also the evolutionary dynamics.

Definition 2. An equilibrium consists of an equilibrium distribution of types represented by a CDF F^* and PDF f^* on $[0, 1]$ and an equilibrium strategy $\sigma_\theta^* \in [0, 1]$ such that

1. For each $\theta_i \in [0, 1]$,

$$\sigma_{\theta_i}^* \in \arg\max_{\sigma\in[0,1]} \int_0^1 \left\{ \sigma[\sigma_{\theta_j}^*(\theta_i R + (1-\theta_i)r - c) + (1-\sigma_{\theta_j}^*)R] + (1-\sigma)r \right\} f^*(\theta_j)d\theta_j.$$

2. π_θ equalizes across all $\theta \in [0, 1]$, where

$$\pi_{\theta_i} = \int_0^1 [\sigma_{\theta_i}^* \sigma_{\theta_j}^* (\frac{1}{2}R + \frac{1}{2}r - c) + \sigma_{\theta_i}^*(1-\sigma_{\theta_j}^*)R + (1-\sigma_{\theta_i}^*(\theta_j))r] f^*(\theta_j)d\theta_j.$$

By the following argument, we will see that overconfident players will always make up more than half of the population in the stable equilibrium. There is some cutoff type who is indifferent between Fight and Not Fight. Let's call the type θ^*. Any type $\theta > \theta^*$ player plays Fight and any type $\theta < \theta^*$ player always plays Not Fight. Let p^* denote the proportion of type $\theta > \theta^*$ players, Δ^* proportion of type θ^* players, and $1 - \Delta^* - p^*$ proportion of type $\theta < \theta^*$ players.

When a θ player chooses to fight, his expected utility is

$$u_\theta = (p^* + \Delta^*\sigma^*)[\theta R + (1-\theta)r - c] + (1 - p^* - \Delta^*\sigma^*)R$$

It must hold for θ^*, $u_{\theta^*} = r$.

$$(p^* + \Delta^*\sigma^*)[\theta^* R + (1-\theta^*)r - c] + (1 - p^* - \Delta^*\sigma^*)R = r$$

Second, the fitness level of all surviving confidence types must be the same, we must have

$$(p^* + \Delta^*\sigma^*)(\frac{1}{2}R + \frac{1}{2}r - c) + (1 - p^* - \Delta^*\sigma^*)R = r.$$

Therefore, $\theta^* = 1/2$.

The equation simplifies to

$$p^* + \Delta^*\sigma^* = \frac{R - r}{\frac{1}{2}(R-r) + c}.$$

Since $c > R - r$, $p^* + \Delta^*\sigma^* > 2/3$. If there is no atom at $\theta^* = 1/2$, then at least $2/3$ of the population have overconfident beliefs.

There is no non-trivial evolutionary dynamics. Rational players all best respond by mixing between Fighting and Not Fighting. The rational players and overconfident players get the same expected payoffs because the rational players mix. The extremely under-confident players who do not Fight at all are driven out of the market over time, but anyone who mixes between Fighting and Not Fighting survives the market.

6 Conclusion

This paper provides evolutionary justifications for overconfident beliefs. We show that in a class of resource-fighting games, under different settings of observability of players' beliefs, players with overconfident beliefs are more likely to survive and thrive.

Two comments are in order to defend our simple assumptions. First, we only consider players of the same ability in the model. One justification for this is that players of lower abilities will be driven out of the market and competition prior to any belief heterogeneity. Belief heterogeneity only plays a significant role when players of the same ability compete. Second, we also try to be agnostic about equilibrium selection and restrictions on evolutionary stability. Although we have multiple equilibria and unidentified distributions of confidence in the general model, the main point that majority of players are overconfident in equilibrium holds with very little assumption.

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