A Marriage-Market Perspective of the College Gender Gap

Hanzhe Zhang*

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Abstract

This paper shows how women's relatively higher career cost can explain why in most of the developed countries women go to college at a higher rate than men and earn less on average. I assume men and women make costly college and career investments but women face an extra cost for career investment because such investment occurs during their fertile period. The extra career cost discourages women from investing in career but surprisingly encourages more women than men to go to college through a general-equilibrium marriage-market channel that results in an endogenously higher college marriage premium for women.

Keywords: gender-differential career cost, college gender gap, college marriage premium

JEL: C78, D1, I2

*Zhang: Department of Economics, Michigan State University, 486 West Circle Drive, 110 Marshall-Adams Hall, East Lansing, MI 48824-1038, hanzhe@msu.edu. The latest version of the paper is available at http://ssrn.com/abstract=2770854. I am grateful to Gary Becker, Phil Reny, Pierre-André Chiappori, Scott Kominers, and Hugo Sonnenschein for guidance and advice. I benefited from suggestions from Jon Eguia, James Heckman, William Hubbard, Corinne Low, Arijit Mukherjee, Larry Samuelson, Aloysius Siow, Richard van Weelden, and Alessandra Voena. Comments at the Chicago-Northwestern Family Economics Workshop, University of Chicago, Drexel University, Queen's University, Michigan State University, University of Michigan H2D2 Research Day, and Boston College are appreciated. Financial support of the Women and Leadership Archives is also appreciated.

1 Introduction

In the United States, 57% of college students are women but women's unadjusted average income is 78% of men's. More women than men go to college in 26 of 28 countries in the European Union but women's unadjusted average income is less than men's in all 28 countries.

This paper proposes one unifying gender difference to simultaneously explain women's relatively higher college enrollment and relatively lower average income. To the best of my knowledge, no paper has done so.[1] I assume men and women make costly college and career investments but women face an extra cost for career investment because such investment occurs during their fertile period. This biologically rooted gender difference has been used since Siow (1998) to explain economic and social gender differences including the gender pay gap, but has never been used to explain the college gender gap.

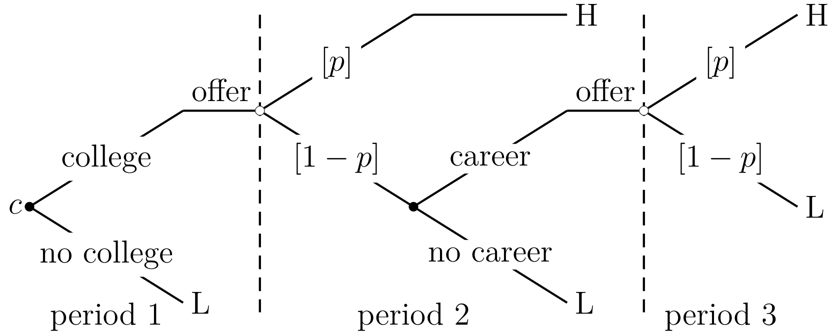

Figure 1: Individual college and career decisions and outcomes.

My model can be simply described as follows. Men and women make college and career decisions to improve their income and marriage prospects (Figure 1). One can pay an individual-specific cost c to attend college in the first period to get a high income with a gender-specific probability p in the second period. A person who fails to get a high income after college can make a career investment in the second period to get a high income with the same probability p in the third period. The only gender difference: career investment costs a man c but costs a woman $c + k$. Therefore, for men, career investment has the same income and marriage benefits and the same cost as college investment so all men who invest in college would also invest in career if college fails. In contrast, for women, career investment has the same benefits as college investment but a higher cost so not all women who invest in college would invest in career if college fails.

[1]Separate factors have been proposed to explain these two gender gaps. Goldin et al. (2006) and Becker et al. (2010a,b) explain the college gender gap using women's higher non-cognitive skills. Iyigun and Walsh (2007) and Chiappori et al. (2009) focus on the marriage market. Adda et al. (2017) explain the gender pay gap using interruptions and lost opportunities associated with child-bearing.

The main result: when investment cost distributions, success probabilities, income premiums, and marriage surplus are gender-symmetric but women face an extra career cost, *more* women than men go to college and fewer women than men earn a high income.

If the *marriage premium*, the difference between a high-income's and a low-income's marriage payoffs, were also fixed to be gender-symmetric, the same number of men and women would go to college and fewer women than men would invest in career, resulting in fewer high-income women than high-income men. However, the marriage premium is endogenously determined. Because there are fewer high-income women than high-income men, high-income women are more valuable than high-income men and the marriage premium is endogenously higher for women than for men. Because of women's higher marriage premium, more women than men go to college. Hence, the extra career cost dis-incentivizes intermediate-ability women from investing in career but indirectly incentivizes lower-ability women to go to college through the general-equilibrium marriage-market channel. The gender pay gap continues to hold because there must be (weakly) fewer high-income women than high-income men to sustain women's higher marriage premium.

Sections 2 and 3 describe the model and its unique equilibrium. Section 4 presents the main result. Section 5 discusses testable implications and comparative statics. The appendix includes omitted details and omitted proofs.

2 Model

There is an infinite number of discrete periods. At the beginning of each period, men and women reach adulthood and make decisions in the next three periods (ages 18-24, 25-31 and 32-40). They are endowed with heterogeneous investment costs c distributed according to continuous and strictly increasing distributions F_m and F_w on $[0, \overline{c}]$.

2.1 College and Career Investments

In the first period, each agent decides whether or not to go to college. One who does not go to college earns a low lifetime income and enters the marriage market, and one who goes to college pays the cost c and delays entering the marriage market.[2]

In the second period, each college investor receives either a high-income offer with probability p_m if a man and p_w if a woman or a low-income offer. A person who receives a high-income offer will accept it and enter the marriage market. A person who receives a

[2] Investing and entering the marriage market in the same period can be shown to be weakly dominated by investing and delaying entrance to the marriage market.

low-income offer will decide whether or not to make a career investment, which could be working hard on the current job, searching for a better job, or receiving more training or education. One who does not make a career investment earns a low income and enters the marriage market, and one who makes a career investment skips the marriage market. The only gender difference in the model is in career investment cost:

Assumption 1. *The career investment costs a man c but costs a woman $c + k$.*

In the third period, each career investor receives either a high lifetime income with probability p_m if a man and p_w if a woman or a low lifetime income, and enters the marriage market regardless of the outcome.

Assume every agent of the same gender-cost type chooses the same strategy so that the strategies are stationary. $\sigma_{m1}(c)$ represents the probability that a cost c man invests in college in the first period, and $\sigma_{m2}(c)$ represents the probability that a cost c man invests in career in the second period. For example, $(\sigma_{m1}(c), \sigma_{m2}(c)) = (1, 0)$ represents that a cost c man invests in college but does not invest in career. $\sigma_{w1}(c)$ and $\sigma_{w2}(c)$ are similarly defined.

2.2 Income Distributions

Investment strategies $\sigma_m = (\sigma_{m1}, \sigma_{m2})$ and $\sigma_w = (\sigma_{w1}, \sigma_{w2})$ *induce* income distributions $G_m = (G_{mH}, G_{mL})$ and $G_w = (G_{wH}, G_{wL})$. High-income men include those who receive a high income after college and enter the marriage market in the second period of their life as well as those who make a career investment after college fails and enter the marriage market with a high income in the third period of their life:

$$G_{mH} = \int_0^{\bar{c}} \sigma_{m1}(c)[p_m + (1 - p_m)\sigma_{m2}(c)p_m]dF_m(c).$$

G_{wH} is similarly characterized, and the induced masses of low-income men and low-income women are simply $G_{mL} = 1 - G_{mH}$ and $G_{wL} = 1 - G_{wH}$. Stationary income distributions G_m and G_w describe both the lifetime income distributions of a generation and the income distributions in a period's overlapping-generations marriage market.

2.3 The Marriage Market

A couple can generate a *marriage surplus* over what they can generate separately on their own using their incomes. The surplus is represented by s_{HH}, s_{HL}, s_{LH}, or s_{LL}. Assume the surplus is strictly increasing in incomes, and

Assumption 2. *The surplus is strictly supermodular: $s_{HH} - s_{HL} > s_{LH} - s_{LL}$.*

In the marriage market men and women frictionlessly match and bargain over the division of the surplus until a *stable outcome* is reached. A stable outcome consists of a *stable matching* and *stable marriage payoffs*. Stable matching $G = (G_{HH}, G_{HL}, G_{LH}, G_{LL})$ describes the different masses of couples of different income combinations subject to feasibility constraints $(G_{HH} + G_{HL} \leq G_{mH}$, etc.). Stable marriage payoffs satisfy three conditions.

1. (*Individual rationality*) Every person receives at least as much as being single: $v_{m\tau_m} \geq 0$ and $v_{w\tau_w} \geq 0$, for any $\tau_m, \tau_w \in \{H, L\}$.

2. (*Pairwise efficiency*) Every matched couple divides the entire surplus: $v_{m\tau_m} + v_{w\tau_w} = s_{\tau_m\tau_w}$ if $G_{\tau_m\tau_w} > 0$.

3. (*No blocking pair*) The surplus any man and any woman generate if married to each other does not exceed the sum of their payoffs: $v_{m\tau_m} + v_{w\tau_w} \geq s_{\tau_m\tau_w}$ for any $\tau_m \in \text{support}(G_m)$ and $\tau_w \in \text{support}(G_w)$.[3]

A stable outcome always exists by Theorem 2 in Gretsky et al. (1992).[4]

2.4 Payoffs

u_{mH} is a single high-income man's payoff, and u_{mL}, u_{wH} and u_{wL} are similarly defined. v_{mH} is a married high-income man's marriage payoff, and v_{mL}, v_{wH} and v_{wL} are similarly defined. A person derives utility from single payoff u and marriage payoff v as well as dis-utility from investment costs. Assume agents are risk-neural and do not discount. Section A.1 presents a household model consistent with the single payoffs, surplus monotonicity, surplus supermodularity, and transferable utility. For expositional ease, I define *income premiums* $\Delta u_m \equiv u_{mH} - u_{mL}$ and $\Delta u_w \equiv u_{wH} - u_{wL}$ and *marriage premiums* $\Delta v_m \equiv v_{mH} - v_{mL}$ and $\Delta v_w \equiv v_{wH} - v_{wL}$, and refer to $p_m \Delta u_m$ and $p_w \Delta u_w$ as *income gains* and $p_m \Delta v_m$ and $p_w \Delta v_w$ as *marriage gains*.

[3] The induced income distributions may not have full support. For example, when no man goes to college, there is no high-income man, so stability conditions do not restrict the out-of-support high-income men's marriage payoff. I assume for any out-of-support τ_m, $v_{m\tau_m} \equiv \max_{\tau_w \in \text{support}(G_w)}(s_{\tau_m\tau_w} - v_{w\tau_w})$, and for any out-of-support τ_w, $v_{w\tau_w} \equiv \max_{\tau_m \in \text{support}(G_m)}(s_{\tau_m\tau_w} - v_{m\tau_m})$. The assumption is for the sake of completeness. When everyone maximizes utility, the situation with missing types will not arise in this model, as there is always a positive mass of agents with investment costs sufficiently close to zero willing to make investments.

[4] I describe a matching by four numbers G_{HH}, G_{HL}, G_{LH}, and G_{LL}, and I say two matchings are equivalent if the four numbers are the same. I do not need to define the exact pairwise matching in the marriage market with a continuum of agents because people make investment decisions based only on their (expected) stable marriage payoffs.

3 Equilibrium

Definition 1. $(\sigma_m^*, \sigma_w^*, G_m^*, G_w^*, G^*, v_m^*, v_w^*)$ *is an* equilibrium *if*

1. *Investment strategy $\sigma_m^*(c)$ maximizes each cost c man's expected utility when men's marriage payoff is v_m^*, and investment strategy $\sigma_w^*(c)$ maximizes each cost c woman's expected utility when women's marriage payoff is v_w^*.*

2. *Men's income distribution G_m^* is induced by men's investment strategy σ_m^*, and women's income distribution G_w^* is induced by women's investment strategy σ_w^*.*

3. *(G^*, v_m^*, v_w^*) is a stable outcome of the marriage market (G_m^*, G_w^*).*

In the remainder of this section, I characterize each equilibrium component and prove equilibrium existence and uniqueness.

3.1 Optimal College and Career Investments

Lemma 1. *Suppose Assumption 1 holds and fix marriage premiums Δv_m and Δv_w. Men with a cost below $c_m \equiv p_m(\Delta u_m + \Delta v_m)$ invest in college and in career. Women with a cost below $c_w \equiv p_w(\Delta u_w + \Delta v_w)$ invest in college, and women with a cost below $c_w - k$ invest in career.*

Men's optimal investments are solved by backward induction. A man who invests in career after college fails pays an investment cost c and expects an income gain $p_m \Delta u_m$ and a marriage gain $p_m \Delta v_m$, so he makes a career investment if and only if the investment cost is lower than the total income and marriage gains: $c \leq p_m \Delta u_m + p_m \Delta v_m = c_m$. A college investment incurs the same cost and yields the same expected gain as a career investment. Therefore, only cost $c \leq c_m$ men, the ones who would invest in career if needed, invest in college.

Women's optimal investments are also solved by backward induction. A woman who invests in career after college fails pays a cost $c + k$ and expects an income gain $p_w \Delta u_w$ and a marriage gain $p_w \Delta v_w$. Therefore, only women with a cost smaller than $p_w \Delta u_w + p_w \Delta v_w - k = c_w - k$ make a career investment. Women's gain from a college investment is the same as from a career investment, but the cost is lower from a college investment. Hence all women with cost below c_w invest in college; women with cost between $c_w - k$ and c_w invest in college but would not invest in career.

Note that Assumption 2 is not needed for the cutoff characterization of the optimal investments in Lemma 1, because agents base their investment decisions only on marriage premiums Δv_m and Δv_w. Also note that, since the two college cutoffs c_m and c_w both equal the total income and marriage gains, when the exogenous income gains and endogenous marriage gains are fixed to be the same for the two genders, the two college cutoffs equalize. In the main result, more women go to college under the setting that is gender-symmetric except for the career cost: $c_w > c_m$. Since income premiums and success probabilities are fixed to be gender-symmetric, Δv_w must be endogenously higher than Δv_m for the college gender gap to hold in equilibrium.

3.2 Induced Income Distributions

Lemma 2. *When agents play the optimal investment strategies characterized by cutoff costs c_m and c_w in Lemma 1, the induced income distributions are*

$$\begin{aligned} G_{mH} &= F_m(c_m)p_m + F_m(c_m)(1-p_m)p_m = F_m(c_m)(2-p_m)p_m, \\ G_{wH} &= F_w(c_w)p_w + F_w(c_w - k)(1-p_w)p_w, \end{aligned}$$

$G_{mL} = 1 - G_{mH}$, *and* $G_{wL} = 1 - G_{wH}$.

Depending on the parameters, the mass of high-income men could be more than, equal to, or less than that of high-income women. For example, if the cutoffs c_m and c_w and the distributions F_m and F_w are the same, more men than women would earn a high income, because the same number of men and women would go to college but fewer women than man would invest in career. Stable marriage market outcomes are characterized differently under the three cases, as follows.

3.3 Stable Marriage Market Outcome

Since the marriage surplus is assumed to be strictly supermodular in incomes, by the standard result of Becker (1973), stable matching is strictly positive assortative in incomes, and men and women rank by their incomes and match accordingly.

Lemma 3. *Suppose Assumption 2 holds. Figure 2 shows possible stable matchings.*

1. *When $G_{mH} > G_{wH}$ (Market 1), there is mass G_{wH} of (H, H) couples, mass $G_{mH} - G_{wH}$ of (H, L) couples, and mass $1 - G_{mH}$ of (L, L) couples.*

2. *When $G_{mH} = G_{wH} \equiv G_H$ (Market 2), there is mass G_H of (H, H) couples and mass $1 - G_H$ of (L, L) couples.*

3. When $G_{mH} < G_{wH}$ (Market 3), there is mass G_{mH} of (H, H) couples, mass $G_{wH} - G_{mH}$ of (L, H) couples, and mass $1 - G_{wH}$ of (L, L) couples.

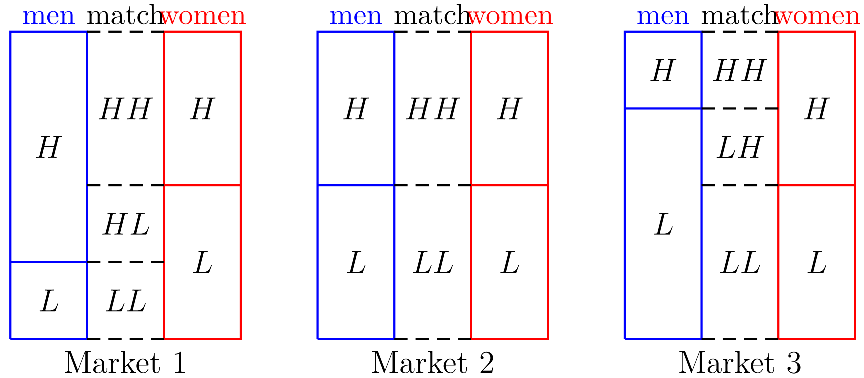

Figure 2: Stable matchings under strict surplus supermodularity.

Finally, I characterize stable marriage payoffs. Since there is an equal mass of men and women, how the low-income couples divide their marriage surplus is indeterminate. In spite of this indeterminacy, we can determine stable marriage premiums which are the only variables needed for agents to decide their optimal investments in Lemma 1. First, notice from Figure 2 that there are always (H, H) and (L, L) couples, so the pairwise efficiency conditions $v_{mH} + v_{wH} = s_{HH}$ and $v_{mL} + v_{wL} = s_{LL}$ always hold. Following the two conditions, Δv_w is uniquely determined as $s_{HH} - s_{LL} - \Delta v_m$.

It remains to determine Δv_m. In Market 1 in Figure 2, when $G_{mH} > G_{wH}$, both high-income and low-income men could marry low-income women, so men's marriage premium is simply the difference between the surplus a high-income man generates and the surplus a low-income man generates with the same low-income woman: $\Delta v_m = s_{HL} - s_{LL}$. In Market 3, when $G_{mH} < G_{wH}$, men's marriage premium is similarly determined to be the difference between the surpluses a man generates with a high-income woman: $\Delta v_m = s_{HH} - s_{LH}$. The marriage premium in Market 2 can be any value between the two extremes $s_{HL} - s_{LL}$ and $s_{HH} - s_{LH}$.

Lemma 4. *Suppose Assumption 2 holds. Stable marriage payoffs are determined up to a constant: $v_{mL} + v_{wL} = s_{LL}$. Men's stable marriage premium is*

$$\Delta v_m = \begin{cases} s_{HL} - s_{LL} & \text{if } G_{mH} > G_{wH} \\ \lambda(s_{HH} - s_{LH}) + (1-\lambda)(s_{HL} - s_{LL}), \lambda \in [0,1] & \text{if } G_{mH} = G_{wH} \\ s_{HH} - s_{LH} & \text{if } G_{mH} < G_{wH}. \end{cases}$$

Women's stable marriage premium is

$$\Delta v_w = s_{HH} - s_{LL} - \Delta v_m.$$

These stable marriage premiums reflect men and women's relative values in the marriage market. Namely, when husband and wife contribute symmetrically to the marriage surplus ($s_{HL} = s_{LH}$) and there are more high-income men than high-income women in the marriage market ($G_{mH} > G_{wH}$), women's stable marriage premium is higher than men's ($\Delta v_w > \Delta v_w$). Moreoever, when the mass of high-income men decreases relative to that of high-income women, men's stable marriage premium increases and women's decreases. This inverse relationship captures the basic economic principle increasing scarcity creates more value. Furthermore, as men's stable marriage premium increases, more men end up with a high income; but as more men earn a high income, men's stable marriage premium decreases. This value monotonicity facilitates the proof of equilibrium uniqueness.

3.4 Equilibrium Existence and Uniqueness

Theorem 1. *There exists a unique equilibrium: investment strategies, income distributions, matching, and marriage premiums are uniquely determined.*

4 Main Result

Proposition 1. *Suppose the setting is gender-symmetric except for the career cost ($F_m = F_w$, $p_m = p_w$, $\Delta u_m = \Delta u_w$, and $s_{HL} = s_{LH}$, but $k > 0$). Strictly more women than men go to college and weakly fewer women than men earn a high income.*

Proof. Let $F_m = F_w \equiv F$, $p_m = p_w \equiv p$, $\Delta u_m = \Delta u_w \equiv \Delta u$.

First, suppose by contradiction that weakly more men than women go to college in equilibrium: $F(c_m^*) \geq F(c_w^*)$. $c_m^* = p\Delta u + p\Delta v_m^* \geq p\Delta u + p\Delta v_w^* = c_w^*$ implies $\Delta v_m^* \geq \Delta v_w^*$. As a result,

$$\begin{aligned} G_{mH}^* &= F(p\Delta v_m^* + p\Delta u)p(2-p) \\ &= F(p\Delta v_m^* + p\Delta u)p + F(p\Delta v_m^* + p\Delta u)p(1-p) \\ &> F(p\Delta v_w^* + p\Delta u)p + F(p\Delta v_w^* + p\Delta u - k)p(1-p) = G_{wH}^*. \end{aligned}$$

When $G_{mH}^* > G_{wH}^*$, the stable marriage premiums are $\Delta v_m^* = s_{HL} - s_{LL}$ and $\Delta v_w^* = s_{HH} - s_{HL}$ by Lemma 4. Since $s_{HL} = s_{LH}$, $\Delta v_m^* = s_{HL} - s_{LL} = s_{LH} - s_{LL} < s_{HH} - s_{HL} = \Delta v_w^*$

where the inequality follows Assumption 2. $\Delta v_m^* < \Delta v_w^*$ contradicts the earlier conclusion that $\Delta v_m^* \geq \Delta v_w^*$.

Second, suppose by contradiction that strictly more women than men earn a high income in equilibrium: $G_{mH}^* < G_{wH}^*$. The inequality implies by Lemma 4 that the stable marriage premiums are $\Delta v_m^* = s_{HH} - s_{LH}$ and $\Delta v_w^* = s_{LH} - s_{LL}$. Since $s_{HL} = s_{LH}$, $\Delta v_m^* = s_{HH} - s_{LH} = s_{HH} - s_{HL} > s_{LH} - s_{LL} = \Delta v_w^*$, where the inequality follows Assumption 2. However, if $\Delta v_m^* > \Delta v_w^*$, there cannot be strictly more high-income women than high-income men, because

$$
\begin{aligned}
G_{wH}^* &= F(p\Delta v_w^* + p\Delta u)p + F(p\Delta v_w^* + p\Delta u - k)p(1-p) \\
&< F(p\Delta v_m^* + p\Delta u)p + F(p\Delta v_m^* + p\Delta u)p(1-p) \\
&= F(p\Delta v_m^* + p\Delta u)p(2-p) = G_{mH}^*.
\end{aligned}
$$

Hence a contradiction with the premise $G_{mH}^* < G_{wH}^*$.
QED □

When the setting is gender-symmetric except for the career cost and the marriage premiums are also fixed to be same for the two genders, the same number of men and women would go to college and fewer women than men would earn a high income. However, because fewer women than men earn a high income, the women who earn a high income are scarcer and more valuable than the men who achieve the same feat. Women's marriage premium is endogenously higher than men's. Women's higher marriage premium prompts more women to attend college and particularly alters the decisions of the women on the margin of college investment. In short, the extra career cost directly discourages some women from making career investments during their fertile years but also indirectly encourages other women to make college investments through the general-equilibrium marriage market channel.

Previous papers have explained the college gender gap using gender differences in non-cognitive skills (Goldin et al., 2006; Becker et al., 2010a,b), in skill premiums (Dougherty, 2005; Hubbard, 2011), and in family roles (Iyigun and Walsh, 2007; Chiappori et al., 2009, 2016). Proposition 1 shows that more women than men could go to college without any of these gender differences, or by extension, even with some gender differences that still constrain women from college investments. The result does not invalidate any of the previous explanations, as adding other gender differences would only reinforce the result. The result merely suggests that there could be a common force that governs a phenomenon common in over seventy countries around the world. At the same time, the gender pay gap is also maintained without additional gender differences, a result unattainable with the previous explanations.

Two key assumptions underlying the main result are gender-differential career cost and the endogenous division of strictly supermodular surplus that leads to strategic substitutability of college and career investments.

4.1 Gender-Differential Career Cost

Assumption 1 gender-differential career cost is necessary and sufficient for the gender pay gap and is necessary but not sufficient for the college gender gap. Without any gender difference in costs, the same number of men and women would invest in college, invest in career, and earn a high income. If marriage premiums were fixed to be the same for the two genders, women's higher career cost alone would generate the gender pay gap but would not generate the college gender gap without other gender differences.

The extra career cost k is neither present when women invest in college nor when men invest in college or in career. It intends to capture women's fertility loss when they invest in career and delay marriage and child-bearing during their fertile period. Men remain fertile through the investment stage, because college and career investments take relatively less time compared to the length of time they are fertile, whereas women may become less fertile when they finish investing, because the investments take a significant portion of their relatively shorter fertile period. This biological difference is also reflected in gender-differential mating preferences: men significantly prefer younger women but women do not significantly prefer younger men (Low, 2016).[5] The extra career cost could also reflect other obstacles such as discriminations against women climbing the career ladder and imperfections associated with maternity leave policies.

The consequences of the gender-differential career cost are manifested in gender differences in income and education. Women only started to significantly lag behind in their earnings in their fertile years: women of ages 16-24 and 25-34 in 2005 respectively earned 93% and 89.1% of their male counterparts, but women of ages 35-44 and 45-54 only earned 75.6% and 75.4% of their male counterparts, respectively (Figure 3). Furthermore, advanced education, a form of career investment, also interferes with fertility. In 2012, among women of ages 40-50 who have ever given birth, 54% and 41% of women with master's degree+ and bachelor's degree had children after 30, compared to 22% and 17% for women with two-year degrees or some college and high school or less, and more educated women have had fewer children on average.

[5]The online appendix shows that the main result would not change if women's fertility were modeled as an additional dimension to income in the marriage market and the career cost is endogenously determined as the loss associated with reproductive decline.

4.2 Endogenous Division of Strictly Supermodular Surplus

The extra career cost directly reduces career investments but does not directly increase college investments, because college and career investments are not direct substitutes. The college gender gap would not arise without the general-equilibrium marriage-market setup, namely, endogenous division of marriage surplus. The endogenous surplus division makes college and career investments *strategic substitutes*. Namely, the extra career cost directly reduces the career investment incentives of the intermediate-ability women on the margin of career investments, those women with costs slightly above $c_w^* - k$, and indirectly through the endogenous division of marriage surplus, encourages the women who are on the margin of college investments, those lower-ability women with costs slightly below c_w^*.

However, endogenous division of an arbitrary surplus is not enough. Assumption 2 strict surplus supermodularity is necessary. If the surplus is not strictly supermodular, $s_{HH}+s_{LL} \leq s_{HL} + s_{LH}$, the stable matching would be negative assortative, so there would be positive masses of (H, L) and (L, H) couples and the pairwise efficiency conditions $v_{mH}^* + v_{wL}^* = s_{HL}$ and $v_{mL}^* + v_{wH}^* = s_{LH}$ would yield $\Delta v_m^* - \Delta v_w^* = s_{HL} - s_{LH} = 0$ when $s_{HL} = s_{LH}$. Without strict surplus supermodularity, the same number of men and women would go to college and invest in career, so there is neither a college gender gap nor a gender pay gap. Strict surplus supermodularity in incomes is theoretically supported by a household model involving public good provision in Section A.1 and is empirically supported by a myriad of evidence on positive assortative matching in incomes and educations in the United States and other developed countries (Blossfeld and Timm, 2003; Schwartz and Mare, 2005; Stevenson and Wolfers, 2007; Greenwood et al., 2014, 2016).

The counterintuitive result that the disadvantaged women can invest more fundamentally stems from the endogenous surplus division. Other assumptions, multiple investments, multi-dimensional marriage types as shown in the online appendix, and gender balance, are not necessary.[6] In this model, the purpose of imposing the cost on career investments rather than directly on college investments is to derive the clean result that more women always go to college. Section A.3 presents a two-period investment-and-marriage model in which there is only one investment and women succeed with probability $p < 1$ but men succeed with probability 1. Even though lower success probability directly reduces women's investment incentives, more women could end up investing due to the endogenous surplus division in the marriage market.

[6] The main result would not change if there is a gender imbalance because the marriage premiums would not change. Some minor results change: if there are more men than women, some low-income men would remain single and the division of the marriage surplus between the low-income couples would be determinate, $v_{mL} = 0$ and $v_{wL} = s_{LL}$, so the marriage payoffs would be uniquely determined.

5 Testable Implications and Comparative Statics

5.1 Testable Implications

One key testable implication of the model is women's higher marriage premium. There are two more testable implications of the model about gender differences in *marginal* net gains to college and *average* net gains to college.

Proposition 2. *Suppose the setting is gender-symmetric except for the career cost.*

(a) Women's marriage premium is higher than men's.

(b) Net gains to college are higher for near-marginal *college-investing women than for men of the same abilities.*

(c) Net gains to college could be high or lower on average *for college-investing women than for college-investing men.*

5.1.1 Marginal Net Gains to College

A *near-marginal* college-investing woman does not make a career investment, so her equilibrium net gains to college are simply $p_w \Delta u_w + p_w \Delta v_w^* - c$. In contrast, a near-marginal college-investing man has positive net gains from career,

$$p_m \Delta u_m + p_m \Delta v_m^* - c + (1 - p_m)[p_m \Delta u_m + p_m \Delta v_m^* - c].$$

However, for men of the same abilities as near-marginal college-investing women, they either do not invest in college (for cost $c \in [c_m^*, c_w^*)$), or they have very small net gains from a career investment (for cost c close to c_m^*). Hence, a near-marginal college-investing woman has higher net gains to college than a man of the same ability because of endogenously higher marriage premium.

5.1.2 Average Net Gains to College

A career-investing woman's net gains to college include an extra career cost,

$$p_w \Delta u_w + p_w \Delta v_w^* - c + (1 - p_w)[p_w \Delta u_w + p_w \Delta v_w^* - c - k].$$

Although women's marriage premium is higher than men's, because of the extra career cost, it is unclear whether their net gains to college are higher. Although some near-marginal college-investing women have higher net gains to college than their male counterparts, the infra-marginal college-investing and career-investing women may have lower net gains to college. Because of the heterogeneity, it is unclear whether women or men on average have net gains to college.

5.2 Comparative Statics

Figure 3 shows the changes in the college gender gap and the gender pay gap in the United States from 1979 to 2005. Women have not only caught up with but surpassed men in college. Women's college enrollment rate more than doubled from 20.3% in 1970 to 44.1% in 2010, while men's stagnated around 30-35%. Meanwhile, the pay gap has also been closing for each age group. For example, the female-to-male income ratio for 25- to 34-years-olds increased from 67.4% in 1979 to 89.1% in 2005. These patterns of reversed college gender gap and closing gender pay gap are also found throughout the world. Only five countries had more women than men in college in 1970, but 67 countries achieved the same feat in 2010. Although across the EU economy women earned on average 16.4% less than men in 2014, they earned on average 27.5% less than men in 2001.

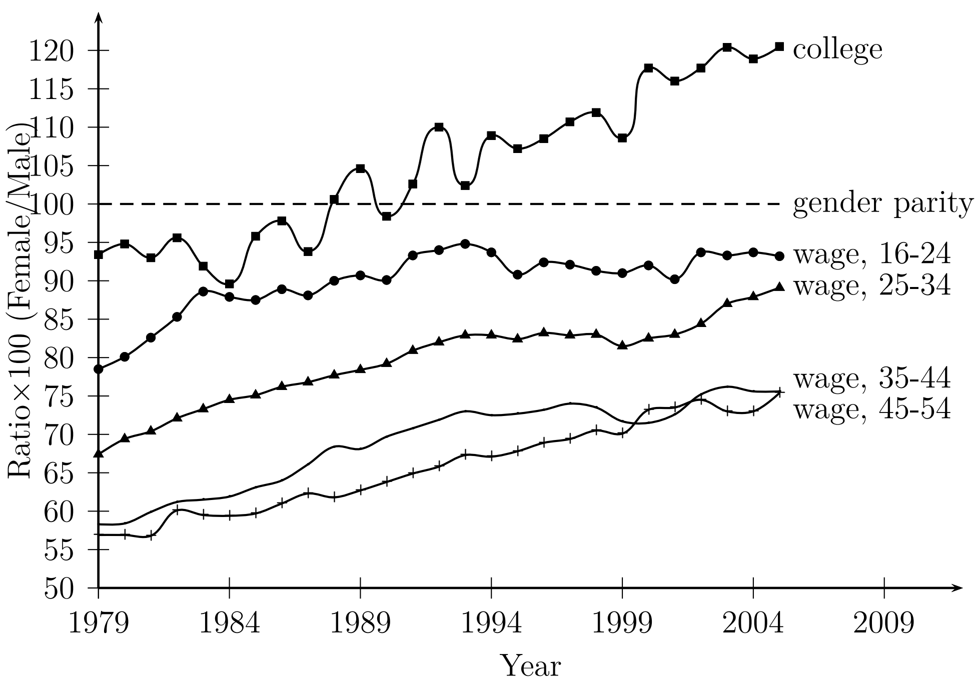

Figure 3: College Gender Gap and Gender Pay Gaps by Age Groups, 1979-2005.

Comparative statics results identify contributing factors to the rapid rises in women's earnings and education: (a) declining human capital investment costs, (b) improved labor-market opportunities and increased college and skill income premiums and (c) improved marriage prospects of high-income women. Most notably, contrary to the previous predictions, Proposition 3(d) predicts a narrowing rather than widening college gender gap in the future when the extra career cost decreases.

Proposition 3. *Suppose that there are strictly fewer high-income women than high-income men in equilibrium before and after parameters change. The college gender gap and the gender pay gap both shrink when*

(a) women's cost distribution F_w decreases first-order stochastically dominatedly,

(b) women's success probability p_w and income premium Δu_w increase, and

(c) women's marriage premium $\Delta v_w^* = s_{HH} - s_{HL}$ increases.

Women's college enrollment weakly decreases *and women's average income strictly increases when*

(d) women's additional career cost k decreases.

5.2.1 Decrease in Investment Costs

Human capital investment costs include psychic costs, social stigma against women going to college and pursuing career dreams, tuition costs, and opportunity costs. First, psychic costs have decreased asymmetrically for women. As more jobs required skilled workers who could complete complex and interpersonal tasks, non-cognitive skills and social skills have been increasingly valued over physical abilities, and women have been shown to perform better than men in key measures of college success such as high school grades, attendance rates, and externalizing behavior (Goldin et al., 2006; Becker et al., 2010a; Deming, 2017). Second, the social stigma against women going to college and working lessened. Socially, the feminism movement in the 1960s awakened women to search for their identity and more actively participate in the labor force (Friedman, 1977). Technologically, the introduction of household items such as dishwashers, washing machines, and the internet reduced the need for women to stay at home and encouraged them to seek more work and education in preparation for work (Greenwood et al., 2014). Finally, although tuition costs and opportunity costs increased, these increases should not have affected the gender gaps for the following reasons: first, the increases in returns on education more than offset the increase in tuition costs and forgone earnings; second, the change in tuition cost did not differ by gender, so the change should not have affected the gender gaps; third, the percent of students affected by budget constraint only 8% according to Carneiro and Heckman (2002).

5.2.2 Increases in Labor Force Participation and Income Premium

Women's labor force participation rate has drastically increased. The rate among college women has increased from 40% in 1960 to close to 80% in 2010, and the rate among non-college women has also increased, while men did not experience any drastic change (Greenwood et al., 2016). The relaxation of the marriage bar after World War II has notably helped women to prolong their labor market activity past their marriage and to find jobs in the first place (Goldin, 1990). The invention of and legal access to birth control pill have also helped women plan their family and career better (Goldin and Katz, 2002; Bailey, 2006).

Women's income premium has also increased. As the demand for skilled labor increased, the income gap between skilled and unskilled workers has increased (Card and Lemieux, 2001; Dougherty, 2005; Mulligan and Rubinstein, 2008; Hubbard, 2011), and the increase has been gender-asymmetric: changes in the allocation of labor between industries and occupations have strongly favored college graduates and females but not males (Katz and Murphy, 1992; Acemoglu and Autor, 2011). Better amenities and more family-friendly policies such as more flexible work schedules and maternity leave policies also gave women more options in their career choices, reflected in more women venturing into previously male-dominated occupations (Bronson, 2013).

5.2.3 Increase in Marriage Premium

Chiappori et al. (2016) found a significantly increased college marriage premium over the last few decades for women but not for men. Educated women's improving marriage prospects are also supported by their improved marriage partners (Low, 2016; Zhang, 2017), improved material well-being (DiPrete and Buchmann, 2006), and steadily increasing degree of positive assortative matching in income and education since 1950s (Blossfeld and Timm, 2003; Schwartz and Mare, 2005; Stevenson and Wolfers, 2007; Greenwood et al., 2016).

Women's increased marriage premium has been caused by changes within the household. The time spent on household chores by women shifted to time spent on children and time outside the household. According to American Time Use Survey, from 1975 to 2003, for women with a child age 5 or less, paid work increased 85% from 1.55 hours per day to 2.88, child care increased 63% from 1.63 to 2.67, and household work decreased 27% from 3.67 to 2.64; for women with a child age 5 to 17, paid work increased 35% from 2.71 to 3.68, child care increased 74% from 0.65 to 1.13, and household work decreased 22% from 3.63 to 2.83; shopping slightly increased and leisure slightly decreased (Chiappori et al., 2009; Browning et al., 2014).

5.2.4 Decrease in Women's Career Cost

Women's career cost due to delayed marriage and childbearing, despite its lasting presence, has decreased over time as women are expected to have fewer children. From 1976 to 2014, the proportion of American couples wanting three or more children decreased from 59% to 32% while the proportion wanting no child, one child, and two children has increased from 10% to 15%, from 10% to 18%, and from 22% to 35%, respectively (Wang and Parker, 2014). If this gender-differential cost is mitigated or eliminated, the current model predicts a possible re-convergence of the college gender gap, in contrast to previous models that would predict a further widening of the college gender gap. OECD (2008) predicts that the percentage of women among all college students will move up from 57% in 2005 to 62% in 2030 in the United States, from 58% to 64% in Canada, and from 57% to 71% in the United Kingdom. Previous predictions are based on simple extrapolations of existing trends in a partial equilibrium setup. Adding the countervailing general-equilibrium marriage-market effect to the extrapolation of trends should yield more plausible prediction of the changes in the college gender gap.

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A Appendix: Omitted Details and Omitted Proofs

A.1 A Household Model

The following household model is consistent with the set up of the marriage market in the paper. A single man allocates his income y_m between a private good q_m and a public good Q to achieve

$$u_m(y_m) = \max_{q_m+Q\leq y_m} q_m Q = y_m^2/4.$$

Similarly, a single woman allocates her income y_w between a private good q_w and a public good Q to achieve

$$u_w(y_w) = \max_{q_w+Q\leq y_w} q_w Q = y_w^2/4.$$

If the income y_m man and the income y_w woman pool together their income and maximize their joint utility $q_m Q + q_w Q$, their maximized utility would be

$$u(y_m, y_w) = \max_{q_m+q_w+Q\leq y_m+y_w} (q_m + q_w)Q = (y_m + y_w)^2/4.$$

The surplus due to the couple's marriage is

$$s(y_m, y_w) = (y_m + y_w)^2/4 - y_m^2/4 - y_w^2/4 = y_m y_w/2.$$

The surplus is strictly increasing and strictly supermodular in incomes.

The household model is also consistent with transferable utility. To obtain a surplus division $v_m = \lambda y_m y_w/2$ and $v_w = (1-\lambda)y_m y_w/2$, spend $(y_m+y_w)/2$ on Q, $[y_m^2/2+\lambda y_m y_w]/(y_m+y_w)$ on q_m, and $[y_w^2/2+(1-\lambda)y_m y_w]/(y_m+y_w)$ on q_w so that the man gets a payoff $u_m(y_m)+v_m$ and the woman gets a payoff of $u_w(y_w) + v_w$.

More generally, a household model with utility of the form $q^\alpha Q^\beta$, $\alpha + \beta > 1$, yields a surplus function strictly increasing and strictly supermodular in incomes. The individual utilities are

$$u_m(y) = u_w(y) = \max_{q+Q\leq y} q^\alpha Q^\beta = \frac{\alpha^\alpha \beta^\beta}{(\alpha+\beta)^{\alpha+\beta}} y^{\alpha+\beta}.$$

The joint utility $u(y_m, y_w) = \max_{q_m+q_w+Q\leq y_m+y_w} (q_m^\alpha + q_w^\alpha)Q^\beta$

$$\begin{aligned}
&= \max_Q \left[\max_{q_m+q_w\leq y_m+y_w-Q} (q_m^\alpha + q_w^\alpha) \right] Q^\beta \\
&= \max_Q (1_{\alpha\geq 1} + 1_{\alpha<1} 2^{1-\alpha})(y_m + y_w - Q)^\alpha Q^\beta \\
&= (1_{\alpha\geq 1} + 1_{\alpha<1} 2^{1-\alpha}) \frac{\alpha^\alpha \beta^\beta}{(\alpha+\beta)^{\alpha+\beta}} (y_m + y_w)^{\alpha+\beta}.
\end{aligned}$$

The marriage surplus is

$$s(y_m, y_w) = \frac{\alpha^\alpha \beta^\beta}{(\alpha+\beta)^{\alpha+\beta}} \left[(1_{\alpha \geq 1} + 1_{\alpha<1} 2^{1-\alpha})(y_m + y_w)^{\alpha+\beta} - y_m^{\alpha+\beta} - y_w^{\alpha+\beta} \right].$$

A.2 Proof of Theorem 1

All equilibrium components can be uniquely determined by men's marriage premium Δv_m. By Lemma 4, women's marriage premium is $\Delta v_w(\Delta v_m) = s_{HH} - s_{LL} - \Delta v_m$. By Lemma 1, optimal investments facing marriage premiums Δv_m and $\Delta v_w(\Delta v_m)$ are characterized by cutoffs $c_m(\Delta v_m) \equiv p_m \Delta u_m + p_m \Delta v_m$ and $c_w(\Delta v_m) \equiv p_w \Delta u_w + p_w \Delta v_w(\Delta v_m)$. By Lemma 2, the induced masses of high-income men and women are

$$\begin{aligned} G_{mH}(\Delta v_m) &\equiv F_m(c_m(\Delta v_m)) p_m (2 - p_m) \\ G_{wH}(\Delta v_m) &\equiv F_w(c_w(\Delta v_m)) p_w + F_w(c_w(\Delta v_m) - k) p_w (1 - p_w). \end{aligned}$$

Stable matching under $(G_{mH}(\Delta v_m), G_{wH}(\Delta v_m))$ is described by Lemma 3.

Equilibrium existence and uniqueness can be shown under three cases. First, suppose $G_{mH}(s_{HL} - s_{LL}) > G_{wH}(s_{HL} - s_{LL})$, that is, even when men's stable marriage premium is the smallest possible to give men the lowest incentives to invest and women's stable marriage premium is the biggest possible to give women the highest incentives to invest, the induced mass of high-income men is strictly bigger than the induced mass of high-income women. $\Delta v_m^* = s_{HL} - s_{LL}$ and strictly more men than women earn a high income in equilibrium. It is the unique equilibrium: if $\Delta v_m^* > s_{HL} - s_{LL}$, there will be even more high-income men and fewer high-income women compared to the equilibrium above, so $G_{mH} > G_{wH}$ and the marriage premium has to be $s_{HL} - s_{LL}$ in any equilibrium. Second, by the same logic, there is a unique equilibrium when $G_{mH}(s_{HH} - s_{LH}) < G_{wH}(s_{HH} - s_{LH})$: $\Delta v_m^* = s_{HH} - s_{LH}$ and strictly more women than men earn a high income in equilibrium even when women have the lowest possible incentives and men have the highest possible investment incentives. Finally, the third case: $G_{mH}(s_{HL} - s_{LL}) \leq G_{wH}(s_{HL} - s_{LL})$ and $G_{mH}(s_{HH} - s_{LH}) \geq G_{wH}(s_{HH} - s_{LH})$. Equilibrium is characterized by Δv_m^*, the unique solution to $G_{mH}(\Delta v_m) = G_{wH}(\Delta v_m)$; the solution is unique because $G_{mH}(\Delta v_m)$ is strictly increasing and $G_{wH}(\Delta v_m)$ is strictly decreasing. There is an equal mass of high-income men and high-income women in equilibrium. It is the unique equilibrium, because there cannot be strictly more high-income men than high-income women: with the stable marriage premiums under $G_{mH} > G_{wH}$, optimal investments induce $G_{mH}(s_{HL} - s_{LL}) \leq G_{wH}(s_{HL} - s_{LL})$, contradicting $G_{mH} > G_{wH}$; analogously, there cannot be strictly fewer high-income men than high-income women.

QED

A.3 A Two-Period Investment-and-Marriage Model

Men with costs distributed according to F_m can invest to earn a high income for sure, and women with costs distributed according to F_w can invest to earn a high income with probability $p < 1$ and a low income with probability $1 - p$. One who does not invest enters the marriage market as a low-income. The marriage market is organized as in the paper. For simplicity assume everyone only gets a marriage payoff v but no reservation utility u.

Having gone through detailed characterization in the text, we can characterize the equilibrium of this model rather quickly. The equilibrium strategies are characterized by two simple cutoffs $c_m^* = \Delta v_m^*$ and $c_w^* = p\Delta v_w^*$. Mass $F_m(c_m^*)$ of cost $c \leq c_m^*$ men and mass $F_w(c_w^*)$ of cost $c \leq c_w^*$ women invest. In the marriage market, mass $G_{mH}^* = F_m(c_m^*)$ of men and mass $G_{wH}^* = p_w F_w(c_w^*)$ of women earn a high income. $G_{mH}^* < G_{wH}^*$, $G_{mH}^* = G_{wH}^*$, or $G_{mH}^* > G_{wH}^*$. When $G_{mH} < G_{wH}$, the stable marriage premiums are $\Delta v_m = s_{HL} - s_{LL}$ and $\Delta v_w = s_{HH} - s_{HL}$. When $G_{mH} = G_{wH}$, $\Delta v_m = (1-\lambda)(s_{HL} - s_{LL}) + \lambda(s_{HH} - s_{LH})$ and $\Delta v_w = (1-\lambda)(s_{HH} - s_{HL}) + \lambda(s_{LH} - s_{LL})$. When $G_{mH} > G_{wH}$, $\Delta v_m = s_{HH} - s_{LH}$ and $\Delta v_w = s_{LH} - s_{LL}$.

The equilibrium can be any of the three cases below. First, when $F_m(s_{HL} - s_{LL}) > pF_w(p(s_{HH} - s_{HL}))$, in equilibrium, mass $F_m(s_{HL} - s_{LL})$ of men and $F_w(p(s_{HH} - s_{HL}))$ of women invest. Second, when $F_m(s_{HL} - s_{LL}) \leq pF_w(p(s_{HH} - s_{HL}))$ and $F_m(s_{HH} - s_{LH}) \geq pF_w(p(s_{LH} - s_{LL}))$, mass $F_m((1-\lambda^*)(s_{HL} - s_{LL}) + \lambda^*(s_{HH} - s_{LH}))$ of men and mass $F_w(p[(1-\lambda^*)(s_{HH} - s_{HL}) + \lambda^*(s_{LH} - s_{LL})])$ of women invest, where $F_m((1-\lambda^*)(s_{HL} - s_{LL}) + \lambda^*(s_{HH} - s_{LH})) = pF_w(p[(1-\lambda^*)(s_{HH} - s_{HL}) + \lambda^*(s_{LH} - s_{LL})])$. Third, when $F_m(s_{HH} - s_{LH}) < pF_w(p(s_{LH} - s_{LL}))$, in equilibrium, mass $F_m(s_{HH} - s_{LH})$ of men and $F_w(p(s_{LH} - s_{LL}))$ mass of women invest.

I show that even when F_w first-order stochastically dominates F_m and $p < 1$, more women than men may invest in equilibrium. Whenever $F_w(p(s_{HH} - s_{HL})) > F_m(s_{HL} - s_{LL}) > pF_w(p(s_{HH} - s_{HL}))$ (a sufficient but not necessary condition), strictly more women than men invest. For example, when $F_m(c) = F_w(c) = c/2$ for $c \in [0, 2]$, $s_{HH} = 4$, $s_{HL} = s_{LH} = 2$, $s_{LL} = 1$, and $p = 2/3$, $F_w(p(s_{HH} - s_{HL})) = (2(4-2)/3)/2 = 2/3 > F_m(s_{HL} - s_{LL}) = (2-1)/2 = 1/2 > pF_w(p(s_{HH} - s_{HL})) = (2/3)(2(4-2)/3)/2 = 4/9$. Mass $1/2$ of men and mass $2/3$ of women invest, despite women's disadvantage. For another example in which women's cost distribution is first-order stochastically dominant, $F_m(c) = 1/2$ for $c \in [0, 2]$ and $F_w(c) = 1/2.5$ for $c \in [0, 2.5]$, and the rest are the same as before, $s_{HH} = 4$, $s_{HL} = s_{LH} = 2$, $s_{LL} = 1$, and $p = 2/3$. Then $F_w(p(s_{HH} - s_{HL})) = (2(4-2)/3)/2.5 = 2/3 \times 4/5 = 8/15 >$

$F_m(s_{HL}-s_{LL}) = (2-1)/2 = 1/2 > pF_w(p(s_{HH}-s_{HL})) = (2/3)(2(4-2)/3)/2.5 = 4/9\times 4/5 = 16/45$. Mass 1/2 of men and mass $8/15 > 1/2$ of women invest even though women on average pay a higher investment cost and have a lower chance of succeeding from the investment.

A.4 Proof of Proposition 3

When the mass of high-income women is strictly less than the mass of high-income men, equilibrium marriage premiums are $\Delta v_m^* = s_{HL} - s_{LL}$ and $\Delta v_w^* = s_{HH} - s_{HL}$. Equilibrium college gender gap $F_w(c_w^*) - F_m(c_m^*)$ is

$$F_w(p_w \Delta u_w + p_w(s_{HH} - s_{HL})) - F_m(p_m \Delta u_m + p_m(s_{HL} - s_{LL})).$$

Gender pay gap $[G_{mH}^* u_{mH} + (1 - G_{mH}^*)u_{mL}] - [G_{wH}^* u_{wH} + (1 - G_{wH}^*)u_{wL}]$ is

$$u_{mL} - u_{wL} + \Delta u_m F_m(p_m(s_{HL} - s_{LL}) + p_m \Delta u_m)p_m(2 - p_m) - \Delta u_w[F_w(p_w (s_{HH} - s_{HL}) + p_w \Delta u_w)p_w + F_w(p_w(s_{HH} - s_{HL}) + p_w \Delta u_w - k)p_w(1 - p_w)]$$

College gender gap increases and gender pay gap decreases when (a) F_w decreases first-order stochastically, (b) p_w and $y_{wH} - y_{wL}$ increase, and (c) $s_{HH} - s_{HL}$ increases.

Now I prove (d). When there are unequal masses of high-income men and high-income women, change in k does not change $F_w(c_{w1}^*)$. When there are equal masses of high-income men and high-income women, Δv_w^* is the unique solution to

$$\begin{aligned} &F_m(p_m(s_{HH} - s_{LL} - \Delta v_w^*) + p_m \Delta u_m)p_m(2 - p_m) \\ &\quad = F_w(p_w \Delta v_w^* + p_w \Delta u_w)p_w + F_w(p_w \Delta v_w^* + p_w \Delta u_w - k)p_w(1 - p_w). \end{aligned}$$

By the implicit function theorem,

$$-f_m(c_m^*)p_m^2(2 - p_m)\frac{d\Delta v_w^*}{dk} = f_w(c_w^*)p_w^2 \frac{d\Delta v_w^*}{dk} + f_w(c_w^* - k)p_w^2(1 - p_w)(\frac{d\Delta v_w^*}{dk} - 1).$$

Rearrange,

$$\frac{d\Delta v_w^*}{dk} = \frac{f_w(c_w^* - k)p_w^2(1 - p_w)}{f_m(c_m^*)p_m^2(2 - p_m) + f_w(c_w^*)p_w^2 + f_w(c_w^* - k)p_w^2(1 - p_w)} > 0.$$

Finally,

$$\frac{dF_w(c_w^*)}{dk} = f_w(c_w^*)p_w \frac{d\Delta v_w^*}{dk} > 0.$$

QED