Human Capital Investments and the Marriage Market

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Motivation

- Over 65% American high school graduates go to college.
- Over 80% Americans expect to marry some point in their lives.
- Education and marriage influence each other, and others'.
- Human capital investments and the marriage market are rarely studied in an equilibrium framework.

This Paper

1. Constructs a dynamic equilibrium investments-and-matching framework
   - Equilibrium existence
   - Equilibrium uniqueness
2. Derives new implications about the labor market and the marriage market
   - College investments patterns, e.g. the college gender gap
   - Marriage-age patterns, e.g. relationships with incomes
   - Optimal timing of marriage and investments

Literature

- Static matching: Shapley and Shubik (1972, IJGT), Becker (1973, JPE), Gretsky et al. (1992, ET)
- Investments-and-matching: Cole et al. (2001, JET), Peters and Siow (2002, JPE), Iyigun and Walsh (2007, REStud), Nöldeke and Samuelson (2014, Ecta), Zhang (2015)
- The college gender gap puzzle: Goldin et al. (2006, JEP), Chiappori et al. (2009, AER), Becker et al. (2010, AER)
- Marriage-age patterns: Keeley (1977, EI), Keeley (1979, IER), Bergstrom and Bagnoli (1993, JPE), Bergstrom and Schoeni (1996, JPopE)
- Gender difference in reproductive fitness: Siow (1998, JPE), Díaz-Giménez and Giolito (2013, IER), Low (2015)

Environment

- There are an infinite number of periods.
- At the beginning of each period, unit masses of men and women are born with fixed heterogeneous abilities $\theta_m \sim F_m$, $\theta_w \sim F_w$ on $[0, 1]$.
- Each agent lives for three periods, is risk-neutral, and discounts by δ.
- Agent i's utility, $i = m$ or w, is

$$\begin{aligned} & \text{income } Y_i \text{ in the labor market (LM)} \\ + \quad & \text{marriage paoff } V_i \text{ in the marriage market (MM)} \\ - \quad & \text{investments costs } C_i \end{aligned}$$

College and Career Investments (σ_m, σ_w)

1. Age 1 i decides whether to invest in college or not
   1.1 No college: enters LM and MM, earns Y_{i1} from P_{i1}
   1.2 College: skips LM and MM, pays cost C_{i1}

2. After college, age 2 i receives an offer Y_{i2} from $P_{i2}(\cdot|\theta_i)$
   - $P_{i2}(\cdot|\theta_i)$ FOSDs $P_{i2}(\cdot|\theta_i')$ if $\theta_i > \theta_i'$

   2.1 Accept: enters LM and MM, earns Y_{i2}
   2.2 Reject (career investment): skips LM and MM, pays cost C_{i2}

3. After career investment, age 3 i receives Y_{i3} from $P_{i3}(\cdot|\theta_i)$
   - $P_{i3}(\cdot|\theta_i)$ FOSDs $P_{i3}(\cdot|\theta_i')$ if $\theta_i > \theta_i'$

- $\sigma_{i1}(\theta_i)$ and $\sigma_{i2}(\theta_i, Y_{i2})$ probabilities of investments.
- (σ_m, σ_w) stationary and symmetric strategies.

The Marriage Market (μ_m, μ_w)

- Age 3 woman's fitness $R \sim \Phi$ on $[R_L, R_H]$; others R_H.
- Overlapping generations in the marriage market each period.
- Measures μ_m on $[Y_{mL}, Y_{mH}]$ and μ_w on $[Y_{wL}, Y_{wH}] \times [R_L, R_H]$.
- σ_i induces measure μ_i, $i = m, w$; e.g. $\mu_m([Y_{mL}, Y_m]) =$

$$\int (1 - \sigma_{m1}(\theta_m)) P_{m1}(Y_m) dF_m(\theta_m) +$$
$$\int \int_{Y_{mL}}^{Y_m} [1 - \sigma_{m2}(\theta_m, Y_{m2})] \, dP_{m2}(Y_{m2}|\theta_m) dF_m(\theta_m) +$$
$$\int \int \sigma_{m2}(\theta_m, Y_{m2}) P_{m3}(Y_m|\theta_m) dP_{m2}(Y_{m2}|\theta_m) dF_m(\theta_m).$$

- Marriage surplus $S(Y_m, Y_w, R)$ strictly increasing, continuous.

Stable Outcome (μ, V_m, V_w)

A *stable outcome* of the marriage market (μ_m, μ_w) consists of

- *a stable matching* μ
  - $\mu(\mathcal{Y}_m, \mathcal{Y}_w, \mathcal{R})$: measure of $(Y_m, Y_w, R) \in (\mathcal{Y}_m, \mathcal{Y}_w, \mathcal{R})$ couples
  - μ has marginals μ_m and μ_w
- *stable marriage payoff functions* V_m and V_w
  1. Every agent gets weakly more than staying single,
  $$V_m(Y_m), V_w(Y_w, R) \geq 0 \quad \forall Y_m, Y_w, R,$$
  2. Every couple divides the entire marriage surplus,
  $$V_m(Y_m) + V_w(Y_w, R) = S(Y_m, Y_w, R) \quad \forall (Y_m, Y_w, R) \in \text{supp}(\mu),$$
  3. No man and woman can benefit from rematching,
  $$V_m(Y_m) + V_w(Y_w, R) \geq S(Y_m, Y_w, R) \quad \forall Y_m, Y_w, R.$$

Equilibrium

$(\sigma_m^*, \sigma_w^*, \mu_m^*, \mu_w^*, \mu^*, V_m^*, V_w^*)$ is an *equilibrium* iff

1. σ_m^* maximizes each man's expected utility given V_m^*, and σ_w^* maximizes each woman's expected utility given V_w^*,

2. σ_m^* induces μ_m^* and σ_w^* induces μ_w^*, and

3. (μ^*, V_m^*, V_w^*) is a stable outcome of the induced marriage market (μ_m^*, μ_w^*).

Men's Optimal Investments σ_m^*

- θ_m rejects Y_{m2} offer and makes a career investment if

$$\delta\mathbb{E}[Y_{m3} + V_m(Y_{m3})|\theta_m] - C_{m2} > Y_{m2} + V_m(Y_{m2})$$

- Optimal career investment characterized by $Y_{m2}(\theta_m)$,

$$\sigma_{m2}^*(\theta_m, Y_{m2}) = \begin{cases} 0 & Y_{m2} \geq Y_{m2}(\theta_m) \\ 1 & Y_{m2} < Y_{m2}(\theta_m) \end{cases}.$$

- θ_m goes to college if $\mathbb{E}[Y_{m1} + V_m(Y_{m1})] <$

$$-C_{m1} + \delta\mathbb{E}\Big[1_{Y_{m2} > Y_{m2}(\theta_m)}(Y_{m2} + V_m(Y_{m2}))$$
$$+1_{Y_{m2} \leq Y_{m2}(\theta_m)}[-C_{m2} + \delta\mathbb{E}(Y_{m3} + V_{m3}(Y_{m3}))]|\theta_m\Big]$$

- Optimal college investment characterized by θ_{m1},

$$\sigma_{m1}^*(\theta_m) = \begin{cases} 1 & \theta_m \geq \theta_{m1} \\ 0 & \theta_m < \theta_{m1} \end{cases}.$$

Women's Optimal Investments σ_w^*

- θ_w rejects Y_{w2} offer and makes a career investment if

$$\begin{aligned} &\delta \mathbb{E}[Y_{w3} + V_w(Y_{w3}, R_H)|\theta_w] - C_{w2} \\ &- \underbrace{\delta \mathbb{E}[V_w(Y_{w3}, R_H) - V_w(Y_{w3}, R)|\theta_w]}_{\text{fitness cost}} \quad > \quad Y_{w2} + V_w(Y_{w2}) \end{aligned}$$

- Optimal career investment characterized by $Y_{w2}(\theta_w)$,

$$\sigma_{m2}^*(\theta_w, Y_{w2}) = \begin{cases} 0 & Y_{w2} \geq Y_{w2}(\theta_w) \\ 1 & Y_{w2} < Y_{w2}(\theta_w) \end{cases}.$$

- Optimal college investment characterized by θ_{w1},

$$\sigma_{w1}^*(\theta_w) = \begin{cases} 1 & \theta_w \geq \theta_{w1} \\ 0 & \theta_w < \theta_{w1} \end{cases}.$$

Marriage Market $(\mu_m^*, \mu_w^*, \mu^*, V_m^*, V_w^*)$

- $\mu_m^*([Y_{mL}, Y_m]) =$

$$F_m(\theta_{m1})P_{m1}(Y_m) + \int_{\theta_{m1}}^{1} P_{m2}(\min\{Y_m, Y_{m2}(\theta_m)\}|\theta_m)dF_m(\theta_m) + \int_{\theta_{m1}}^{1} [1 - P_{m2}(Y_{m2}(\theta_m)|\theta_m)]P_{m3}(Y_m|\theta_m)dF_m(\theta_m).$$

- Gretsky et al. (1992): Stable outcome (μ^*, V_m^*, V_w^*) exists.

Equilibrium Existence

Theorem 1

An equilibrium $(\sigma_m^*, \sigma_w^*, \mu_m^*, \mu_w^*, \mu^*, V_m^*, V_w^*)$ *exists.*

- Define mappings $(\sigma_m^*, \sigma_w^*) = \Gamma_\sigma(V_m^*, V_w^*)$, $(\mu_m^*, \mu_w^*) = \Gamma_\mu(\sigma_m^*, \sigma_w^*)$, and $(V_m^*, V_w^*) \in \Gamma_V(\mu_m^*, \mu_w^*)$.
- An equilibrium exists iff $(V_m^*, V_w^*) \in \Gamma_V \circ \Gamma_\mu \circ \Gamma_\sigma(V_m^*, V_w^*)$.
- Apply Glicksberg's fixed-point theorem on $\Gamma_V \circ \Gamma_\mu \circ \Gamma_\sigma$.
  - Space of (V_m^*, V_w^*) is nonempty, convex, and compact.
  - $\Gamma_V \circ \Gamma_\mu \circ \Gamma_\sigma(V_m^*, V_w^*)$ is nonempty, convex, and compact.
  - $\Gamma_V \circ \Gamma_\mu \circ \Gamma_\sigma$ is upper-hemicontinuous.

Arzelà-Ascoli Theorem

- Space of (V_m^*, V_w^*), $\Gamma_V \circ \Gamma_\mu \circ \Gamma_\sigma(V_m^*, V_w^*)$ compact, Γ_V u.h.c.
- AA: Equicontinuity and uniform boundedness of V_m^n imply compactness of the set of V_m^n.
- Suppose that $Y_m > Y_m'$ and (Y_w^n, R^n) are matched,

$$
\begin{aligned}
& V_m^n(Y_m) - V_m^n(Y_m') \\
= \; & [S(Y_m, Y_w^n, R^n) - V_w^n(Y_w^n, R^n)] \\
& \quad - \max_{(Y_w,R) \in \mathrm{supp}(\mu_w^n)} [S(Y_m', Y_w, R) - V_w(Y_w, R)] \\
\leq \; & [S(Y_m, Y_w^n, R^n) - V_w^n(Y_w^n, R^n)] \\
& \quad - [S(Y_m', Y_w^n, R^n) - V_w^n(Y_w^n, R^n)] \\
= \; & S(Y_m, Y_w^n, R^n) - S(Y_m', Y_w^n, R^n) \\
\leq \; & K(Y_m - Y_m').
\end{aligned}
$$

- Applicable to Chiappori et al. (2009, AER) and Low (2015).

A Simple Model

- No discounting: $\delta = 1$.
- An agent gets a low income Y_{iL} without college.
- An ability θ_i agent gets a high income Y_{iH} with probability θ_i after an investment.
- College and career investments cost the same: $C_i = C_{i1} = C_{i2}$.
- An age 3 woman is R_L with probability ϕ_L and R_H with probability ϕ_H.
- $S(Y_m, Y_w, R)$ is strictly supermodular in Y_m and Y_w, and in Y_m and R.

Equilibrium Investment Cutoffs

- A θ_m man goes to college and makes a career investment if

$$\theta_m > \theta_m^* \equiv \frac{C_m}{Y_{mH} - Y_{mL} + V_m^*(Y_{mH}) - V_m^*(Y_{mL})}.$$

- A θ_w woman goes to college if

$$\theta_w > \theta_{w1}^* \equiv \frac{C_w}{Y_{wH} - Y_{wL} + V_w^*(Y_{wH}, R_H) - V_w^*(Y_{wL}, R_H)}.$$

- A θ_w woman makes a career investment if

$$\theta_w > \theta_{w2}^* \equiv \frac{C_w + \phi_L[V_w^*(Y_{wL}, R_H) - V_w^*(Y_{wL}, R_L)]}{\begin{aligned} Y_{wH} - Y_{wL} + \phi_H[V_w^*(Y_{wH}, R_H) - V_w^*(Y_{wL}, R_H)] \\ + \phi_L[V_w^*(Y_{wH}, R_L) - V_w^*(Y_{wL}, R_L)] \end{aligned}}.$$

Equilibrium Matching

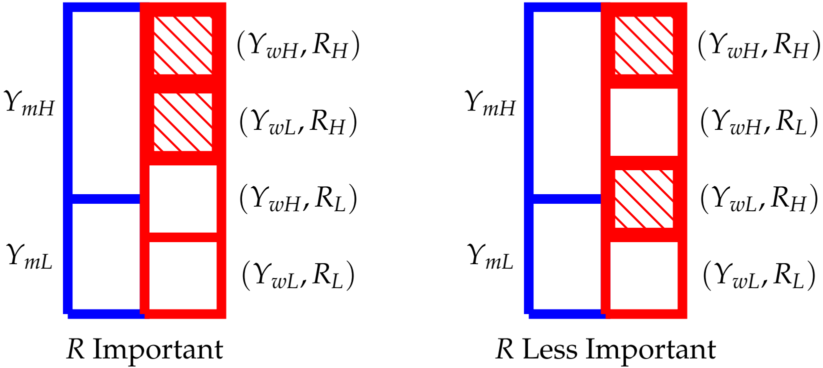


Figure : Stable matchings when the marriage characteristics take binary values.

Equilibrium Uniqueness

Theorem 2

An equilibrium $(\sigma_m^*, \sigma_w^*, \mu_m^*, \mu_w^*, \mu^*, V_m^*, V_w^*)$ *exists uniquely, with the equilibrium marriage payoffs unique up to a constant.*

- $V_m^*(Y_{mH}) - V_m^*(Y_{mL})$ satisfies for some $\lambda \in [0, 1]$

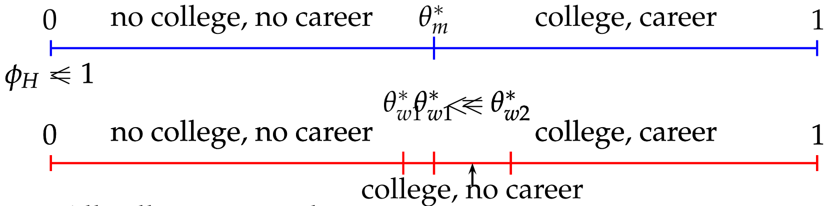
$$\lambda[S(Y_{mH}, Y_{wL}, R_L) - S(Y_{mL}, Y_{wL}, R_L)] \\ +(1-\lambda)[S(Y_{mH}, Y_{wH}, R_H) - S(Y_{mL}, Y_{wH}, R_H)].$$

- λ corresponds to marriage payoffs as well as to a distribution.
  - Bigger $\lambda \Leftrightarrow$ smaller $V_m^*(Y_{mH}) - V_m^*(Y_{mL})$ and more Y_{mH} men.
- The optimal investments to λ induce distribution $\eta(\lambda) \in [0, 1]$.
- λ increases $\Leftrightarrow$ smaller $V_m^*(Y_{mH}) - V_m^*(Y_{mL})$ and more Y_{mH} men $\Rightarrow$ fewer invest $\Rightarrow$ fewer Y_{mH} men $\Leftrightarrow \eta(\lambda)$ increases.
- λ is an equilibrium if $\lambda = \eta(\lambda)$; $\eta(\lambda)$ monotonic $\Rightarrow$ unique λ^*.

The College Gender Gap

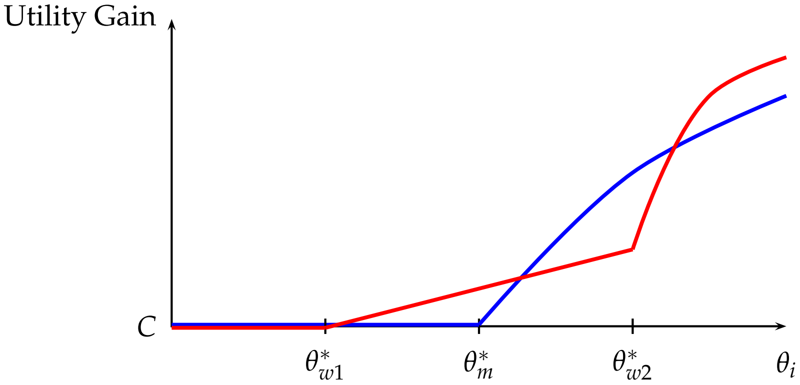
Proposition 1

Suppose setting is gender-symmetric except for fitness; $F_m = F_w$, $C_m = C_w$, $P_{mH} = P_{wH}$, $P_{mL} = P_{wL}$, $S(Y_m, Y_w, R) = S(Y_w, Y_m, R)$, *and* $\phi_H < 1$. *Strictly more women than men go to college.*



- All college men make a career investment.
- Some college women make a career investment.
- Fewer women than men end up with a high income.
- High-income women are scarcer than high-income men.
- College generates higher MM returns for women.

Marginal MM Returns

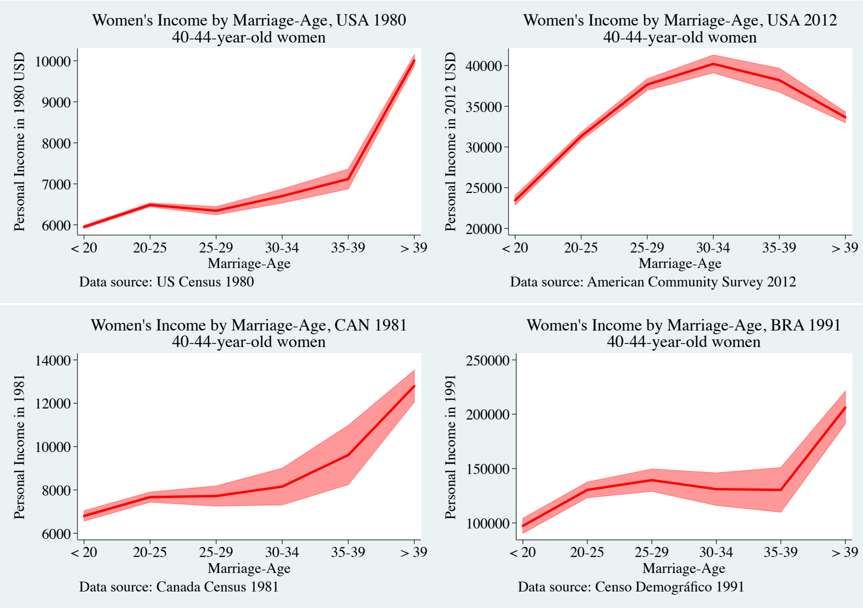



Men's Income by Marriage-Age, USA 1980
40-44-year-old men
Personal Income in 1980 USD
22000
20000
18000
16000
14000
12000
< 20
20-25
25-29
30-34
35-39
> 39
Marriage-Age
Data source: US Census 1980
Men's Income by Marriage-Age, USA 2012
40-44-year-old men
Personal Income in 2012 USD
80000
70000
60000
50000
40000
30000
< 20
20-25
25-29
30-34
35-39
> 39
Marriage-Age
Data source: American Community Survey 2012
Men's Income by Marriage-Age, CAN 1981
40-44-year-old men
Personal Income in 1981
24000
22000
20000
18000
16000
14000
< 20
20-25
25-29
30-34
35-39
> 39
Marriage-Age
Data source: Canada Census 1981
Men's Income by Marriage-Age, BRA 1991
40-44-year-old men
Personal Income in 1991
340000
320000
300000
280000
260000
240000
< 20
20-25
25-29
30-34
35-39
> 39
Marriage-Age
Data source: Censo Demográfico 1991

Men's Income by Marriage-Age

Proposition 2

The relationship between a man's marriage-age and his income in equilibrium is always ***always hump-shaped****: men who marry in the second period have higher average income than those who marry in the first period and those who marry in the third period.*

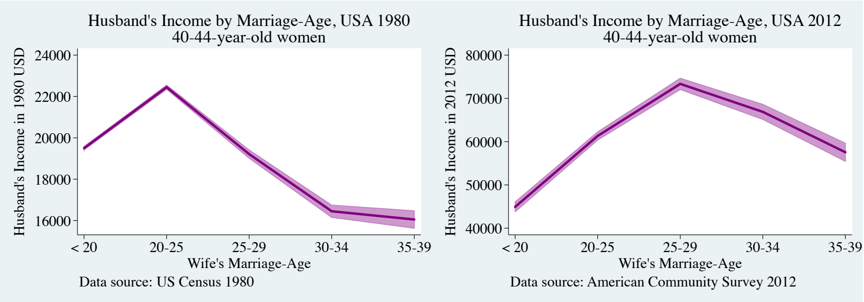

Women's Income by Marriage-Age, USA 1980
40-44-year-old women
Personal Income in 1980 USD
10000
9000
8000
7000
6000
< 20
20-25
25-29
30-34
35-39
> 39
Marriage-Age
Data source: US Census 1980
Women's Income by Marriage-Age, USA 2012
40-44-year-old women
Personal Income in 2012 USD
40000
35000
30000
25000
20000
< 20
20-25
25-29
30-34
35-39
> 39
Marriage-Age
Data source: American Community Survey 2012
Women's Income by Marriage-Age, CAN 1981
40-44-year-old women
Personal Income in 1981
14000
12000
10000
8000
6000
< 20
20-25
25-29
30-34
35-39
> 39
Marriage-Age
Data source: Canada Census 1981
Women's Income by Marriage-Age, BRA 1991
40-44-year-old women
Personal Income in 1991
250000
200000
150000
100000
< 20
20-25
25-29
30-34
35-39
> 39
Marriage-Age
Data source: Censo Demográfico 1991

Women's Income by Marriage-Age

Proposition 3

*The relationship between a woman's marriage-age and her income in equilibrium is either **positive** or **hump-shaped**.*

- *When ϕ_H is sufficiently small and fitness is important in matching, the equilibrium marriage-age/income relationship for women is **positive**: the average income of women increases with their marriage-age.*
- *When ϕ_H is sufficiently large and fitness is less important in matching, the equilibrium relationship is **hump-shaped**: the women who marry in the second period have higher average income than those who marry in the first period and those who marry in the third period.*


Husband's Income by Marriage-Age, USA 1980
40-44-year-old women
Husband's Income in 1980 USD
24000
22000
20000
18000
16000
< 20
20-25
25-29
30-34
35-39
Wife's Marriage-Age
Data source: US Census 1980
Husband's Income by Marriage-Age, USA 2012
40-44-year-old women
Husband's Income in 2012 USD
80000
70000
60000
50000
40000
< 20
20-25
25-29
30-34
35-39
Wife's Marriage-Age
Data source: American Community Survey 2012

Men’s Income by Women’s Marriage-Age

Proposition 4

- *The relationship between a woman’s marriage-age and her husband’s income in equilibrium is* ***always hump-shaped****.*
- *The women who marry in the third period may have* ***higher****-income or* ***lower****-income husbands on average than those who marry in the first period.*
    - *When ϕ_H is sufficiently small and fitness is important, the women who marry in the first period have* ***higher****-income husbands on average than the women who marry in the third period.*
    - *When ϕ_H is sufficiently large and fitness is less important, the women who marry in the first period have* ***lower****-income husbands on average than the women who marry in the third period.*

Postmarital versus Postmarital Investments

Proposition 5

Investing and delaying marriage dominates investing and marrying at the same time for sufficiently patient agents.

- Intuition: marrying early locks a person to a possibly unsuitable partner.

- Explains: 79% of women and 84% of men married after they have finished all schooling.

Conclusion

1. Provides an equilibrium framework with endogenous investments, income, marriage-age, matching, and marriage payoffs.
   - An applicable equilibrium existence result
   - A special equilibrium uniqueness result
2. A unifying theory of labor market and marriage market phenomena
   - The college gender gap puzzle
   - Marriage-age patterns
   - Premarital versus postmarital investments

THANK YOU!

References I

Becker, Gary S., "A Theory of Marriage: Part I," *Journal of Political Economy*, 1973, *81* (4), 813–846.

___ **, William H. J. Hubbard, and Kevin M. Murphy**, "The Market for College Graduates and the Worldwide Boom in Higher Education of Women," *American Economic Review: Papers & Proceedings*, May 2010, *100*, 229–233.

Bergstrom, Theodore C. and Mark Bagnoli, "Courtship as a Waiting Game," *Journal of Political Economy*, February 1993, *101* (1), 185–202.

___ **and Robert F. Schoeni**, "Income Prospects and Age-at-Marriage," *Journal of Population Economics*, 1996, *9*, 115–130.

References II

Chiappori, Pierre-André, Murat Iyigun, and Yoram Weiss, "Investment in Schooling and the Marriage Market," *American Economic Review*, 2009, *99* (5), 1689–1713.

Cole, Harold L., George J. Mailath, and Andrew Postlewaite, "Efficient Non-Contractible Investments in Large Economies," *Journal of Economic Theory*, 2001, *101*, 333–373.

Díaz-Gimémenez, Javier and Eugenio Giolito, "Accounting for the Timing of First Marriage," *International Economic Review*, February 2013, *54* (1), 135–158.

Goldin, Claudia, Lawrence Katz, and Ilyana Kuziemko, "The Homecoming of American College Women: The Reversal of the College Gender Gap," *Journal of Economic Perspectives*, 2006, *20* (4), 133–156.

References III

Gretsky, Neil E., Joseph M. Ostroy, and William R. Zame, "The Nonatomic Assignment Model," *Economic Theory*, 1992, *2*, 103–127.

Iyigun, Murat and Randall P. Walsh, "Building the Family Nest: Premarital Investments, Marriage Markets, and Spousal Allocations," *Review of Economic Studies*, 2007, *74*, 507–535.

Keeley, Michael C., "The Economics of Family Formation: An Investigation of the Age at First Marriage," *Economic Inquiry*, April 1977, *15*, 238–250.

— , "An Analysis of the Age Pattern of First Marriage," *International Economic Review*, June 1979, *20* (2), 527–544.

Low, Corinne, "Pricing the Biological Clock: Reproductive Capital on the US Marriage Market," 2015.

Nöldeke, Georg and Larry Samuelson, "Investment and Competitive Matching," April 2014.

References IV

Peters, Michael and Aloysius Siow, "Competing Premarital Investments," *Journal of Political Economy*, June 2002, *110* (3), 592–608.

Shapley, Lloyd S. and Martin Shubik, "The Assignment Game I: The Core," *International Journal of Game Theory*, 1972, *1* (1), 111–130.

Siow, Aloysius, "Differential Fecundity, Markets, and Gender Roles," *Journal of Political Economy*, April 1998, *106* (2), 334–354.

Zhang, Hanzhe, "Pre-Matching Gambles," 2015.