Courtship as an Investing Game: Labor-Market and Marriage-Market Outcomes by Age at Marriage

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Abstract

This paper documents relationships between age at marriage and labor-market outcome reflected by personal income as well as relationships between age at marriage and marriage-market outcome reflected by spousal income for Americans born from 1900s to 1970s, and motivated by these documented relationships, provides a theory based on human capital investments and differential fecundity to explain these relationships, gender differences in these relationships, as well as the changes of these relationships over time, improving the predictions by the seminal theories of Becker (1973, JPE) and Bergstrom and Bagnoli (1993, JPE).
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1 Introduction

Family is the fundamental building block of the society. Family formation profoundly impacts market activities, household activities, and intergenerational mobility. *When* to form a family, the *marriage timing decision*, is intertwined with many other important life decisions, such as fertility, education, and labor force participation. The issue concerns economists, sociologists, as well as demographers.[1] Becker (1974) wrote in the last section before conclusion in his initial exploration of the theory of marriage market:

> Life-cycle dimensions of marital decisions - for instance, when to marry, how long to stay married, ..., - have received little attention in my earlier paper or thus far in this one. These are intriguing but difficult questions.

He continued,

> A separate paper in the not-too-distant future will develop a more detailed empirical as well as theoretical analysis.

However, to the best of my knowledge, this paper has never materialized, indicating the difficulty and complexity of the underlying problem.

Because any individual's marital decision is influenced by availability of mates in the population, in order to paint a complete picture of the issues related to marriage, we need to incorporate the dynamic marital decisions into the static equilibrium marriage market sketched out in Becker (1973). Moreover, as increasingly more women go to college and participate in the labor force, marital decisions are often associated with human capital investments, so we need to consider the interplay of human capital investment, marriage, and child-bearing.

The interconnectedness of marriage timing and other life decisions is empirically supported. There are systematic relationships between age at first marriage and mid-life personal income. As shown in Figure 1, the relationship for American men born from 1900s to 1970s is *persistently hump-shaped*: those who married in their mid-twenties have the highest average income in their mid-forties, those who married younger or older have a lower average income, and those who stayed unmarried into their forties have the lowest average income. In contrast, the relationship for women has changed over time. As shown in the left panel of Figure 2, for women born before 1940, the relationship was *positive*: the later a woman married, the more she earned on average, and those who stayed unmarried into their forties had the highest average income. As shown in the right panel of Figure 2, for women born after 1940, the relationship became *hump-shaped*: those who married in their twenties had the lowest average income, but those who married in their early thirties earned more than those who married in their late thirties as well as those who stayed unmarried into their forties.[2]

[1] See Glick and Landau (1950) and Oppenheimer (1988) for sociologists' perspectives.

[2] Previous studies documenting the relationships between age at marriage and income include Keeley (1974, 1977, 1979) (American men and women in 1960), Zhang (1995) (Taiwanese men in 1989), Bergstrom and Schoeni (1996) (American men in 1980), and Zhang (2015) (Canadian men and women in 1981 as well as Brazilian men and women in 1991).

Figure 1: Men's Income by Age at Marriage.

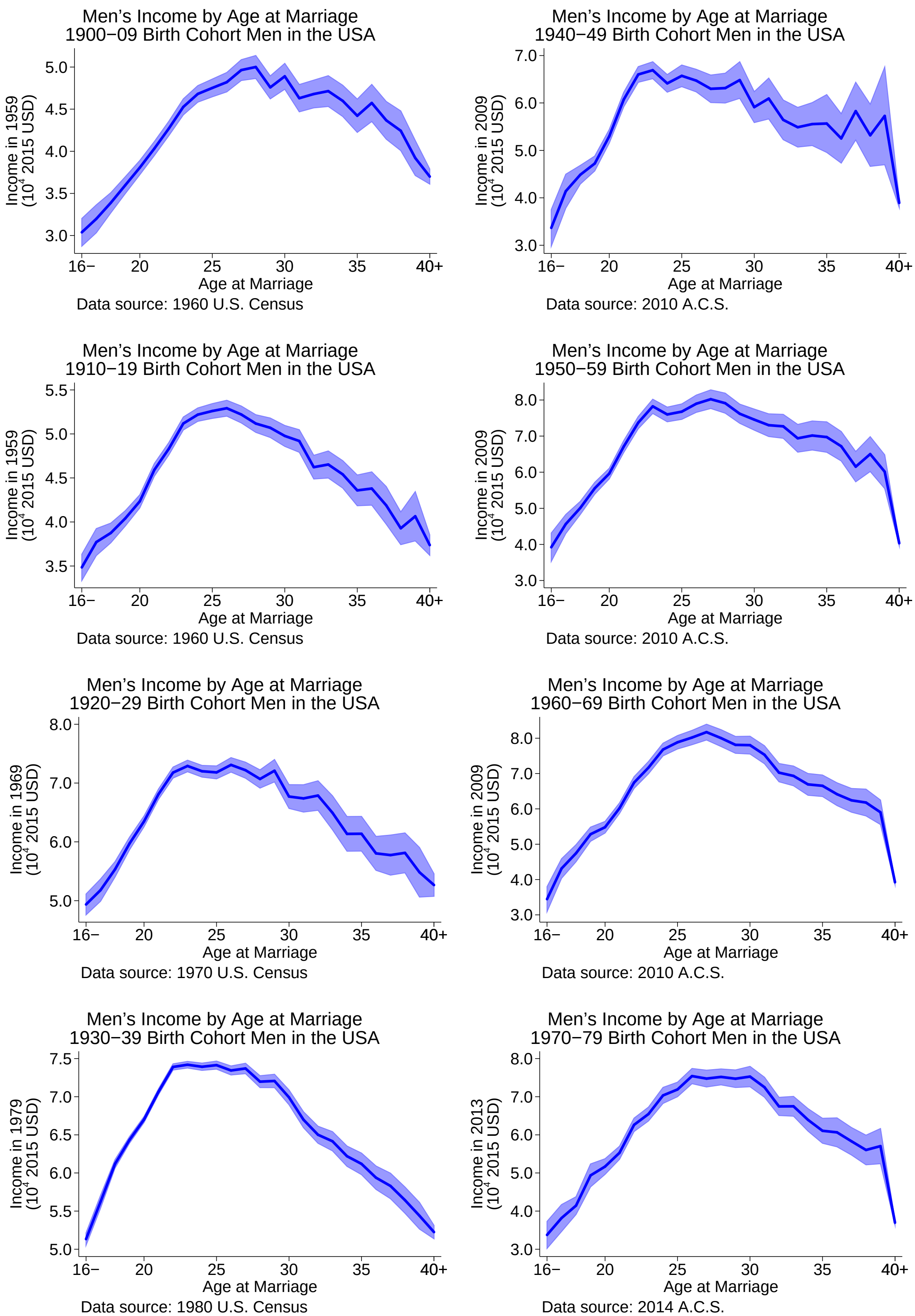
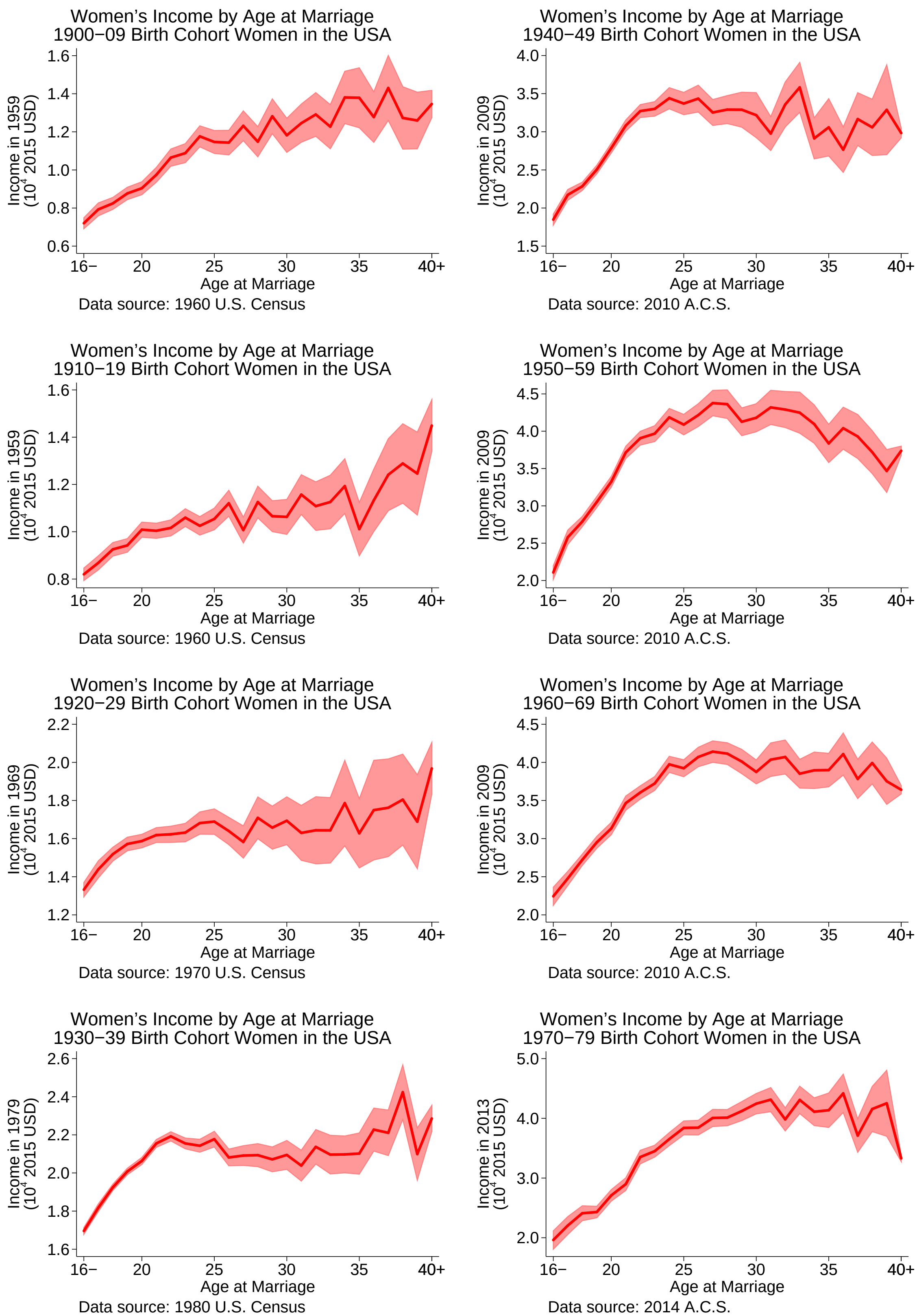


Figure 2: Women's Income by Age at Marriage.



Previous prominent theories, Becker (1973) and Bergstrom and Bagnoli (1993), cannot fully explain the hump shapes in the relationships, gender difference in the relationships, or the changes in the women's relationships over time. Becker (1973) predicts a negative relationship between marriage-age and income for men and a positive relationship for women. Labor specialization in the household is presumed to be the primary purpose of marriage, so higher-income men for their comparative advantage in labor activities and lower-income women for their comparative advantage in household chores gain more from marriage and tend to marry earlier. In contrast, Bergstrom and Bagnoli (1993) predict a positive relationship for men and no relationship for women. They also base their prediction on labor specialization in the household but argue that it takes time for men to reveal their true earning ability but not so for women to reveal their marriage-relevant ability. As a result, capable men wait until they reveal their true earning ability to marry but less capable men marry as early as possible to avoid revealing their true earning ability, thus a positive relationship between marriage-age and income; no woman has any incentive to wait because household ability is immediately revealed in their two-period model so all of them marry in the first period, thus no relationship between women's marriage-age and income.[3]

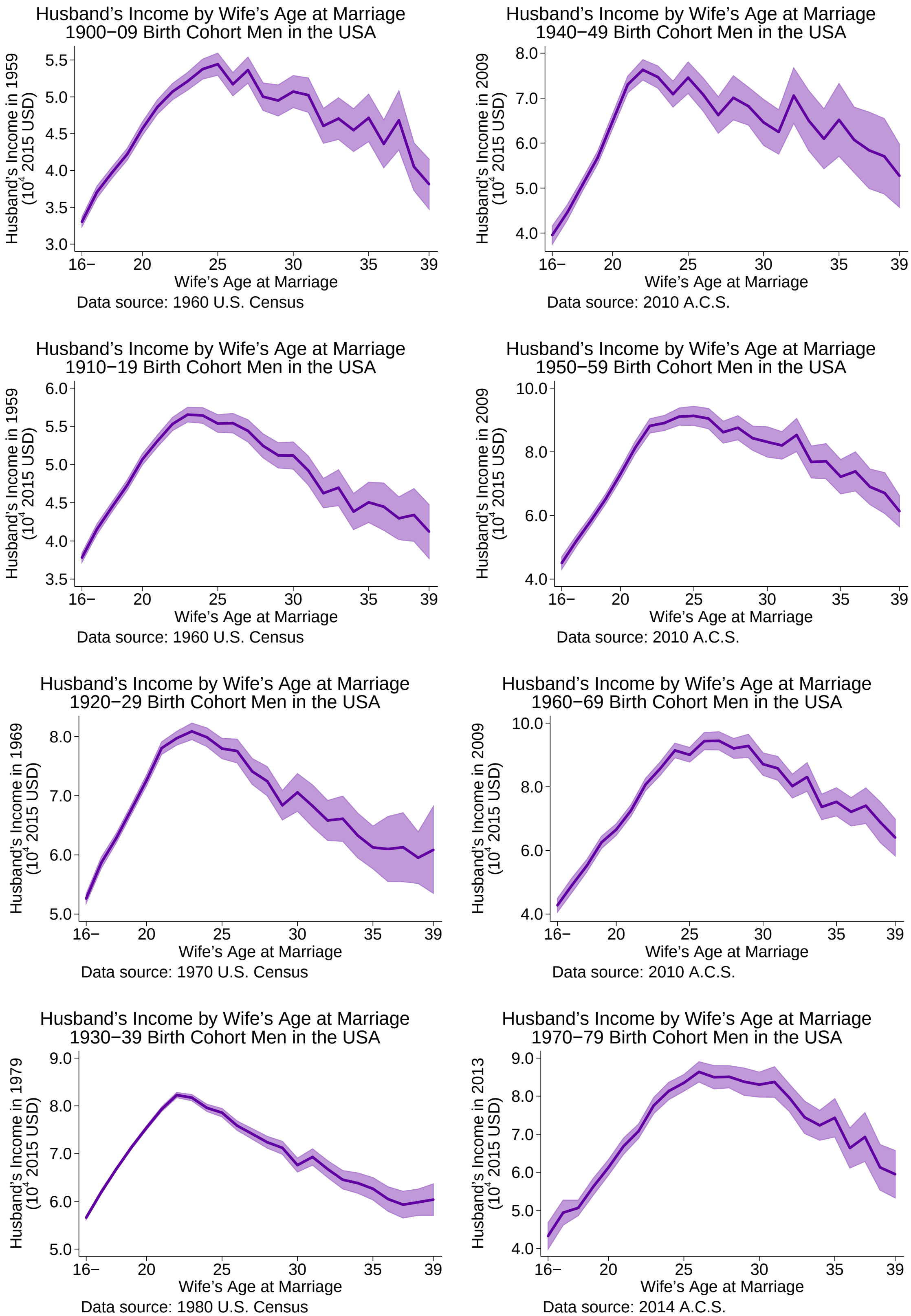
To better explain these relationships and their changes, I provide a theory based on human capital investments and differential fecundity. To explain the hump shapes in the relationships, I assume agents can choose one of three periods to marry and marriage timing is closely tied to their college and career investment decisions. To explain gender differences, I rely on differential fecundity: women have a shorter fertility horizon that interferes with their career investment decisions. Changes in the importance of reproductive fitness relative to income in marriages explain the changes in women's relationships over time, and evidence supports the changing importance of reproductive fitness in the U.S. marriage market.

Furthermore, the new theory offers additional predictions about the relationships between marriage-age and marriage-market outcome that the previous theories cannot explain. The relationship between wife's marriage-age and husband's income (an important indicator of women's marriage-market outcome) has been *persistently hump-shaped* (Figure 3): women who married in the early- to mid-twenties had the highest-income husbands; and it changed from *right-skwed* (left panel of Figure 3) to *left-skewed* (right panel of Figure 3): in the pre-1940 birth cohorts, the women who married in the early twenties had higher-income husbands than those who married in their early thirties; the pattern was reversed for post-1940 birth cohorts: the women who married in the early twenties had lower-income husbands than the women who married in their early thirties. To offer predictions about the matching patterns, I introduce a matching market in which agents are distinguished by their income and reproductive fitness. Changes in the relative importances of income and reproductive fitness in the marriage market explain changes in the observed relationships between wife's marriage-age and husband's income.

Besides these aforementioned relationships, the model also predicts marriage-age distributions and matching by marriage-ages, which I briefly demonstrate with a numerical example in Section 5.

[3] Zhang (1995) provides evidence that both theories are partially correct: marriage-age is negatively related to income among men with nonworking wives, supporting the household specialization effect of Becker (1973), but marriage-age is positively related to income among men with working wives, supporting the revelation effect of Bergstrom and Bagnoli (1993).

Figure 3: Husband's Income by Wife's Age at Marriage.



Two main contributions of this paper are to document the relationships between age at marriage and economic and marital outcomes for almost a century in the United States, and more importantly, to provide a simple equilibrium model simultaneously consistent with all of the documented gender-specific relationships and their changes over time. The model provides a viable path to study more complicated dynamic marital decisions that Becker (1974) has deemed hard to model by embedding the standard equilibrium transferable-utilities marriage market into a more enriched but still tractable framework. The model can predict many patterns because marriage timing, college and career investments, income distributions, who marry whom, as well as the division of marriage surplus are all endogenously determined in its unique equilibrium. The model also has implications about gender differences in college enrollment and earnings, addressed in a separate paper (Zhang, 2017).

One advantage of the paper is to incorporate often ignored yet critical human capital investments into a tractable equilibrium marriage market model. Unlike previous work assuming that each person's income-earning characteristic is exogenously given, this paper provides endogenous determination of human capital investments, division of marriage surplus, and marriage matching. The income and fitness characteristics on which people make their marital decisions are subject to individual choices and idiosyncratic uncertainties.

Another advantage is that the paper only assumes one biologically rooted gender asymmetry - differential fecundity - to explain all the gender-specific patterns and changes. Household specialization and other phenomena such as gender differences in education, income, and age at marriage may arise as a result of the biological difference but are not a priori assumed. This paper with a model with three-period-lived agents refines results in Siow (1998), Chiappori et al. (2009) and Low (2015) that explore marriage-market and labor-market implications of differential fecundity in models with two-period-lived agents. The significance of the biological gender difference in shaping gender differences in social and economic roles is empirically evidential in Adda et al. (2017) and Buckles (2017), for example.

The paper also shows that even without search frictions in the marriage market, key patterns about age at marriage can still arise (Todd et al., 2005; Díaz-Giménez and Giolito, 2013). These results do not undermine the importance of search frictions in explaining marital decisions, but rather highlight the parallel importances of fundamental forces of human capital investments and differential fecundity in explaining age patterns of marriage.

The paper explains many stylized facts qualitatively, and it is far from and has not been intended to be a comprehensive quantitative explanation. In order to achieve a deeper and more complete understanding, other important and realistic factors, such as search frictions and idiosyncratic and couple-specific preference shocks, need to be added into the model. Nonetheless, I hope that the promising results in the paper spark subsequent interests to systematically study labor-market and marriage-market outcomes and their relationships with age at marriage using general-equilibrium dynamic frameworks with premarital investments and transferable-utilities overlapping-generations marriage market.

Section 2 sets up the model and Section 3 characterizes its unique equilibrium. Section 4 shows how the model's predictions are consistent with the documented relationships and changes. Section 5 presents a numerical example to further confirm the predictions and also to show the model's additional predictions on marriage-ages. Section 6 concludes. The appendix includes omitted proofs and additional figures.

2 Model: Courtship as an Investing Game

There is an infinite number of discrete periods. Unit mass of men and unit mass of women become adults each period. They are endowed with heterogenous abilities denoted by $p \in [0, 1]$. These abilities are distributed according to continuous and strictly increasing cumulative distribution functions F_m and F_w. Each person derives utility from income and marriage payoff and disutility from investment costs. Each person is risk-neutral and does not discount.

In the first period of his or her life, each person decides whether or not to go to college. One who does not go to college earns a low income y_{mL}, y_{wL} and enters the marriage market immediately, while one who goes to college incurs a cost c_m, c_w and delays entering the marriage market. In the second period, an ability p person who has gone to college receives a high-income offer y_{mH}, y_{wH} with probability p and accepts it, or receives a low-income offer otherwise and chooses between accepting and rejecting it. One who accepts the offer earns a low income and enters the marriage market, while one who rejects the offer makes a career investment incurring cost c_m, c_w and delays entering the marriage market. In the third period, an ability p person who has made career investment receives a high-income offer with probability p or a low-income offer otherwise, accepts the offer, and enters the marriage market. Figure 4 summarizes these investment and marriage timing decisions.[4]

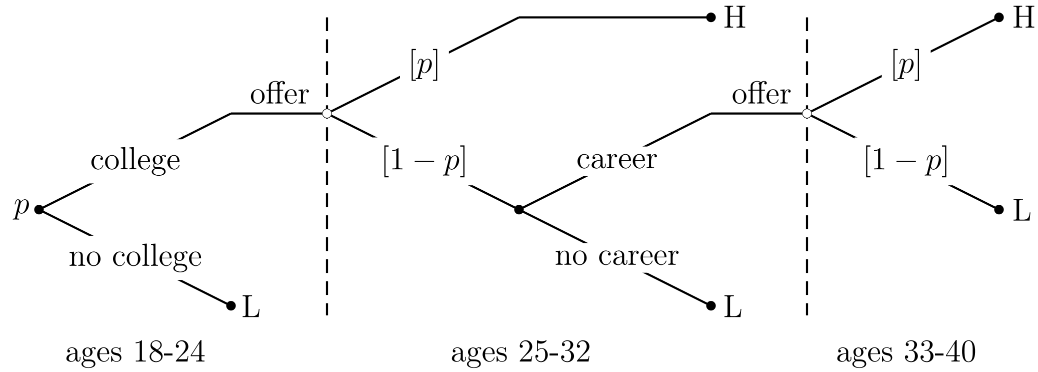


Figure 4: An individual's investment and marriage timing decision.

Assume every person of the same gender-ability type chooses the same strategy so that the strategies are stationary and symmetric. Let $\sigma_{m1}(p)$ be the probability that an ability p man invests in the first period and delays entering the marriage market; σ_{m2}, σ_{w1}, and σ_{w2} are similarly defined. $\sigma_m = (\sigma_{m1}, \sigma_{m2})$ and $\sigma_w = (\sigma_{w1}, \sigma_{w2})$ summarize players' strategies.

[4]Investing and entering the marriage market simultaneously is assumed to be an infeasible strategy, but even if it is a feasible strategy, it can be shown to be weakly dominated by investing and delaying entering the marriage market. In addition, I make an innocuous simplifying assumption that if a person who does not find a mate the first time in the marriage market does not enter the marriage market again. Since the marriage market is stationary, a person who does not find a partner in one period may still not find a partner in the next period when he or she is allowed to enter the marriage market. If he or she does find a partner in the subsequent period, he or she must have taken the place of a person of the same marriage type, and that person becomes unmatched and enters the subsequent period's marriage market. This brings unnecessary complications to the calculation of marriage-age distributions and does not generate additional insights.

The only fundamental gender difference: whereas men are reproductively fit for three periods, women are fit for two periods. A woman who enters the marriage market in the third period stays fit with probability r and becomes less fit with probability $1-r$. Hence, men can be distinguished by income alone (H and L) but women are distinguished by income and fitness (H, L, h, l). The marriage surplus a couple generates is s_{HH}, s_{HL}, s_{Hh}, s_{Hl}, s_{LH}, s_{LL}, s_{Lh}, or s_{Ll}. Assume the surplus increases in income and in fitness, is strictly supermodular in incomes ($s_{HH}+s_{LL}>s_{HL}+s_{LH}$, $s_{Hh}+s_{Ll}>s_{Hl}+s_{Lh}$), and is strictly supermodular in income and fitness ($s_{HH}+s_{Lh}>s_{Hh}+s_{LH}$, $s_{HL}+s_{Ll}>s_{Hl}+s_{LL}$).

Men and women enter the marriage market in different periods based on their abilities, realizations of incomes, and investment decisions, so in each period's marriage market, there are overlapping generations of men and women. The marriage market is summarized by marriage type distributions $G_m=(G_{mH},G_{mL})$ and $G_w=(G_{wH},G_{wL},G_{wh},G_{wl})$, induced by strategies σ_m and σ_w. High-income men include those who accept a high-income offer after college in the second period and those who reject a low-income offer after college and receive a high-income offer in the third period:

$$G_{mH}=\int_0^1 \sigma_{m1}(p)[p+(1-p)\sigma_{m2}(p)p]dF_m(p).$$

The rest of the men are low-income: $G_{mL}=1-G_{mH}$. Similarly, high-income women in the marriage market also include those who get a high-income offer in their second period of life and those who get a high-income offer in their third period of life:

$$G_{wH}=\int_0^1 \sigma_{w1}(p)[p+(1-p)\sigma_{w2}(p)pr]dF_w(p),$$

$$G_{wh}=\int_0^1 \sigma_{w1}(p)(1-p)\sigma_{w2}(p)p(1-r)dF_w(p).$$

Of low-income women, some marry in the first period without going to college, some marry in the second period without making a career investment, and some marry in the third period after a career investment:

$$G_{wL}=\int_0^1 [1-\sigma_{w1}(p)+\sigma_{w1}(p)p+\sigma_{w1}(p)(1-p)\sigma_{w2}(p)(1-pr)]dF_w(p),$$

and $G_{wl}=1-G_{wH}-G_{wL}-G_{wh}$.

In the marriage market, men and women match and bargain over their marriage surplus until a stable outcome is reached. Formally, a *stable outcome* of the marriage market (G_m,G_w) consists of a *stable matching* described by $G_{\tau_m\tau_w}$, $\tau_m\in\{H,L\}$ and $\tau_w\in\{H,L,h,l\}$ such that $\sum_{\tau_w\in\{H,L,h,l\}}G_{\tau_m\tau_w}=G_{m\tau_m}$ for all $\tau_m\in\{H,L\}$ and $\sum_{\tau_m\in\{H,L\}}G_{\tau_m\tau_w}=G_{w\tau_w}$ for all $\tau_w\in\{H,L,h,l\}$; and *stable marriage payoffs* $v_m=(v_{mH},v_{mL})$ and $v_w=(v_{wH},v_{wL},v_{wh},v_{wl})$ such that (1) (individual rationality) every person receives at least as much as staying single: $v_{m\tau_m}$, $v_{w\tau_w}\geq 0$ for all τ_m,τ_w; (2) (pairwise efficiency) every matched couple divides the entire surplus: $v_{m\tau_m}+v_{w\tau_w}=s_{\tau_m\tau_w}$ if $G_{\tau_m\tau_w}>0$; (no blocking pair) no pair of man and woman who are not married to each other can make both of them strictly better off: $v_{m\tau_m}+v_{w\tau_w}\geq s_{\tau_m\tau_w}$. By Gretsky et al. (1992), a stable outcome exists for any marriage market (G_m,G_w).

3 Equilibrium

Definition 1. $(\sigma_m^*, \sigma_w^*, G_m^*, G_w^*, G^*, v_m^*, v_w^*)$ is an *equilibrium* if

1. $\sigma_m^*(p)$ maximizes each ability p man's expected utility rationally expecting v_m^* and $\sigma_w^*(p)$ maximizes each ability p woman's expected utility rationally expecting v_w^*.

2. σ_m^* induces G_m^* and σ_w^* induces G_w^*.

3. (G^*, v_m^*, v_w^*) is a stable outcome of the marriage market (G_m^*, G_w^*).[5]

I subsequently characterize all equilibrium components and prove equilibrium uniqueness.

3.1 Optimal Marriage Timing

Men's optimal investment and marriage timing decisions can be solved by backward induction. When an ability p man fails to capitalize on a college investment, if he invests in career and delays entering the marriage market, he pays a cost c_m and gets an expected income gain of $p(y_{mH} - y_{mL})$ and an expected marriage gain of $p(v_{mH} - v_{mL})$. Therefore, an ability p man makes a career investment if and only if his ability is above

$$p_m \equiv \frac{c_m}{y_{mH} - y_{mL} + v_{mH} - v_{mL}}.$$

An ability p man who decides whether or not to go to college in the first period goes through the same tradeoff. Therefore, all ability $p \geq p_m$ men go to college in the first period and invest in career in the second period if college fails, while all ability $p < p_m$ men avoid investments and enter the marriage market in the first period.

Women's optimal investment and marriage timing decisions can be similarly solved by backward induction but the solution is different because of differential fecundity. If an ability p woman makes a career investment after her college investment fails, her expected income gain is $p(y_{wH} - y_{wL})$, similar to a man, but her expected marriage gain is different: $prv_{wH} + p(1-r)v_{wh} + (1-p)rv_{wL} + (1-p)(1-r)v_{wl} - v_{wL}$. The total expected income and marriage gains are rearranged to be $p(y_{wH} - y_{wL}) + p(v_{wH} - v_{wL}) - (1-r)[p(v_{wH} - v_{wh}) + (1-p)(v_{wL} - v_{wl})]$, where the last term is the extra fertility loss women incur when

[5] The mass of a certain marriage-type induced by certain strategies may be zero. For example, there is no high-income man in the marriage market when every man chooses to marry in the first period, or there is no less-fit woman when no woman chooses to delay marriage into the third period. In such situations, stability conditions do not restrict the out-of-support marriage-type's stable marriage payoff. I assume for any out-of-support $\tau_m \in \{H, L\}$, $v_{m\tau_m} = \max_{\tau_w \in \text{supp}(G_w)}(s_{\tau_m\tau_w} - v_{w\tau_w})$, and for any out-of-support τ_w, $v_{w\tau_w} = \max_{\tau_m \in \text{supp}(G_m)}(s_{\tau_m\tau_w} - v_{m\tau_m})$. Furthermore, I assume that agents have rational expectations and know the stable marriage payoffs if they enter the marriage market as out-of-support marriage-types. When everyone maximizes utility, the situations with missing marriage-types will not arise if the net benefit of investing is sufficiently high. In particular, it suffices to assume $s_{Hl} - s_{Ll} + y_{mH} - y_{mL} > c_m$ and $y_{wH} - y_{wL} > c_w + (1-r)(s_{HL} - s_{Hl})$ so that delaying entrance to the marriage market is ability 1 men's and ability 1 women's strictly dominant strategy in both periods and there are always positive masses of men and women of every marriage type.

they make a career investment. Since a career investment costs c_w, a woman makes a career investment if her ability p is above

$$p_{w2} \equiv \frac{c_w + (1-r)(v_{wL} - v_{wl})}{y_{wH} - y_{wL} + r(v_{wH} - v_{wL}) + (1-r)(v_{wh} - v_{wl})}.$$

An ability p woman who delays marriage from the first period to the second period gets a net benefit of $p(y_{wH} - y_{wL} + v_{wH} - v_{wL}) - c_w$. Therefore, an ability p woman delays marriage from the first period to the second period if and only if her ability is above

$$p_{w1} \equiv \frac{c_w}{y_{wH} - y_{wL} + v_{wH} - v_{wL}}.$$

Note that $p_{w2} > p_{w1}$ when $r < 1$. Therefore, unlike that all men who make a college investment would make a career investment if college fails, some women who make a college investment would not make a career investment regardless of the outcome of the college investment.

Lemma 1. A man makes a college investment in the first period and makes a career investment in the second period after college fails if his ability is above p_m. A woman makes a college investment in the first period if her ability is above p_{w1} and makes a career investment in the second period after college fails if her ability is above p_{w2}.

Because the optimal marriage timing strategies are simply expressed by cutoffs, marriage type distributions can be more simply represented as well.

Lemma 2. The marriage type distributions induced by optimal marriage timing strategies characterized by cutoff abilities p_m, p_{w1}, and p_{w2}, are

$$G_{mH} = \int_{p_m}^{1} p_m(2-p_m)dF_m(p_m), G_{mL} = 1 - G_{mH}, \tag{1}$$

$$G_{wH} = \int_{p_{w1}}^{1} pdF_w(p) + \int_{p_{w2}}^{1} r(1-p)pdF_w(p), , \tag{2}$$

$$G_{wh} = \int_{p_{w2}}^{1} (1-r)(1-p)pdF_w(p), \tag{3}$$

$$G_{wL} = F_w(p_{w1}) + \int_{p_{w1}}^{p_{w2}} (1-p)dF_w(p) + \int_{p_{w2}}^{1} r(1-p)^2 dF_w(p), \tag{4}$$

and $G_{wl} = 1 - G_{wH} - G_{wh} - G_{wL}$.

3.2 Stable Outcome of the Marriage Market

Men and women are matched based on their marriage types. Because the surplus is strictly supermodular in incomes, with the same fitness, a higher-income woman marries a higher-income man. Because the surplus is strictly supermodular in income and fitness, with the same income, a higher-fitness woman marries a higher-income man. However, it is unclear whether a high-income less-fit type h woman or a low-income fit type L woman has a higher-income husband, as this depends on an additional assumption on the surplus. In summary,

Lemma 3. Stable matching satisfies the following properties.

1. Because the surplus is assumed strictly supermodular in incomes, a type H woman marries a higher-income husband than an L woman, and a type h woman marries a higher-income husband than a type l woman.

2. Because the surplus is assumed strictly supermodular in income and fitness, a type H woman marries a higher-income husband than a type h woman, and a type L woman marries a higher-income husband than a type l woman.

3. If $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$, a type h woman marries a higher-income husband than a type L woman. If $s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$, a type h woman marries a lower-income husband than a type L woman.

These properties together are sufficient to determine the stable matching. If $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$, there are seven possible cases of stable matching depending on the relative masses of marriage types, illustrated by markets 1.1-1.7 in the top panel of Figure 5. Alternatively, if $s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$, there are seven other possible cases of stable matching illustrated by markets 2.1-2.7 in the bottom panel of Figure 5. For example, when $G_{mH} < G_{mH} < G_{wH} + G_{wL}$, as illustrated by market 2.3, there is a mass G_{wH} of (H, H) couples, mass $G_{mH} - G_{wH}$ of (H, L) couples, mass $G_{wL} + G_{wh} + G_{wl} - G_{mL}$ of (H, L) couples, mass G_{wh} of (L, h) couples, and mass G_{wl} of (L, l) couples.

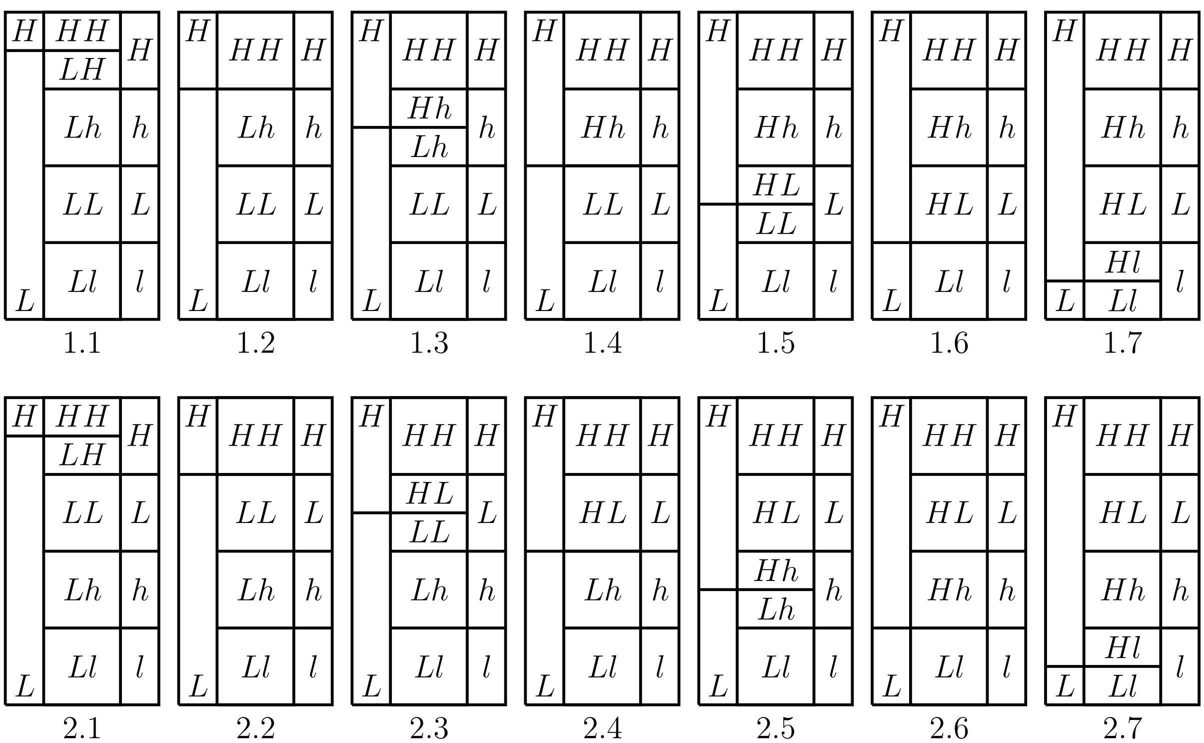


Figure 5: Stable matching.

Lemma 4. Stable marriage payoffs are determined by stable marriage payoff differences summarized in Table 1.[6]

	$v_{mH} - v_{mL}$	$v_{wH} - v_{wL}$	$v_{wh} - v_{wl}$	$v_{wL} - v_{wl}$
1.1	$s_{HH} - s_{LH}$	$s_{LH} - s_{LL}$	$s_{Lh} - s_{Ll}$	$s_{LL} - s_{Ll}$
1.2	$(1-\lambda)(s_{HH} - s_{LH}) + \lambda(s_{Hh} - s_{Lh})$	$(1-\lambda)(s_{LH} - s_{LL}) + \lambda(s_{HH} - s_{Hh} + s_{Lh} - s_{LL})$	$s_{Lh} - s_{Ll}$	$s_{LL} - s_{Ll}$
1.3	$s_{Hh} - s_{Lh}$	$s_{HH} - s_{Hh} + s_{Lh} - s_{LL}$	$s_{Lh} - s_{Ll}$	$s_{LL} - s_{Ll}$
1.4	$(1-\lambda)(s_{Hh} - s_{Lh}) + \lambda(s_{HL} - s_{LL})$	$(1-\lambda)(s_{HH} - s_{Hh} + s_{Lh} - s_{LL}) + \lambda(s_{HH} - s_{HL})$	$(1-\lambda)(s_{Lh} - s_{Ll}) + \lambda(s_{Hh} - s_{Hl})$	$s_{LL} - s_{Ll}$
1.5	$s_{HL} - s_{LL}$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{HL} + s_{LL} - s_{Ll}$	$s_{LL} - s_{Ll}$
1.6	$(1-\lambda)(s_{HL} - s_{LL}) + \lambda(s_{Hl} - s_{Ll})$	$s_{HH} - s_{HL}$	$(1-\lambda)(s_{Hh} - s_{HL} + s_{LL} - s_{Ll}) + \lambda(s_{Hh} - s_{Hl})$	$(1-\lambda)(s_{LL} - s_{Ll}) + \lambda(s_{HL} - s_{Hl})$
1.7	$s_{Hl} - s_{Ll}$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{Hl}$	$s_{HL} - s_{Hl}$
2.1	$s_{HH} - s_{LH}$	$s_{LH} - s_{LL}$	$s_{Lh} - s_{Ll}$	$s_{LL} - s_{Ll}$
2.2	$(1-\lambda)(s_{HH} - s_{LH}) + \lambda(s_{HL} - s_{LL})$	$(1-\lambda)(s_{LH} - s_{LL}) + \lambda(s_{HH} - s_{HL})$	$s_{Lh} - s_{Ll}$	$s_{LL} - s_{Ll}$
2.3	$s_{HL} - s_{LL}$	$s_{HH} - s_{HL}$	$s_{Lh} - s_{Ll}$	$s_{LL} - s_{Ll}$
2.4	$(1-\lambda)(s_{HL} - s_{LL}) + \lambda(s_{Hh} - s_{Lh})$	$s_{HH} - s_{HL}$	$s_{Lh} - s_{Ll}$	$(1-\lambda)(s_{LL} - s_{Ll}) + \lambda(s_{Lh} - s_{Ll} + s_{HL} - s_{Hh})$
2.5	$s_{Hh} - s_{Lh}$	$s_{HH} - s_{HL}$	$s_{Lh} - s_{Ll}$	$s_{Lh} - s_{Ll} + s_{HL} - s_{Hh}$
2.6	$(1-\lambda)(s_{Hh} - s_{Lh}) + \lambda(s_{Hl} - s_{Ll})$	$s_{HH} - s_{HL}$	$(1-\lambda)(s_{Lh} - s_{Ll}) + \lambda(s_{Hh} - s_{Hl})$	$(1-\lambda)(s_{Lh} - s_{Ll} + s_{HL} - s_{Hh}) + \lambda(s_{HL} - s_{Hl})$
2.7	$s_{Hl} - s_{Ll}$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{Hl}$	$s_{HL} - s_{Hl}$

Table 1: Stable marriage payoff differences.

Stable marriage payoff differences are characterized differently under the fourteen different stable matchings. Take an example. When $G_{wH} + G_{wh} < G_{mH} < G_{wH} + G_{wh} + G_{wL}$ (market 1.5 in Figure 5), there are positive masses of (H, H), (H, h), (H, L), (L, L), and (L, l) couples. Since (H, L) couples divide up their surplus, $v_{mH} + v_{wL} = s_{HL}$, and since (L, L) couple divide up their surplus, $v_{mL} + v_{wL} = s_{LL}$. The two conditions together imply $v_{mH} - v_{mL} = s_{HL} - s_{LL}$. Other stable marriage payoff differences are similarly determined: $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, $v_{wh} - v_{wL} = s_{Hh} - s_{HL}$, and $v_{wl} - v_{wL} = s_{Ll} - s_{LL}$. In some situations, market 1.2 for example, stable marriage payoff differences are not uniquely determined, but the indeterminacy will be resolved in equilibrium, as only one set of stable marriage payoff differences can support equilibrium marriage timing decisions.

[6] I ignore the possibility of economically unrealistic and uninteresting cases with oversupply of certain types of men or women, the cases where $G_{mH} > 1$ or $G_{wH} + G_{wh} + G_{wL} > 1$. In these cases, the marriage payoff differences may become 0; for example, for $G_{mH} > 1$, $v_{mH} - v_{mL} = 0$. I stay away from excessive gender imbalances for simplicity and for realistic representation of the economies we are interested in.

3.3 Equilibrium Existence and Uniqueness

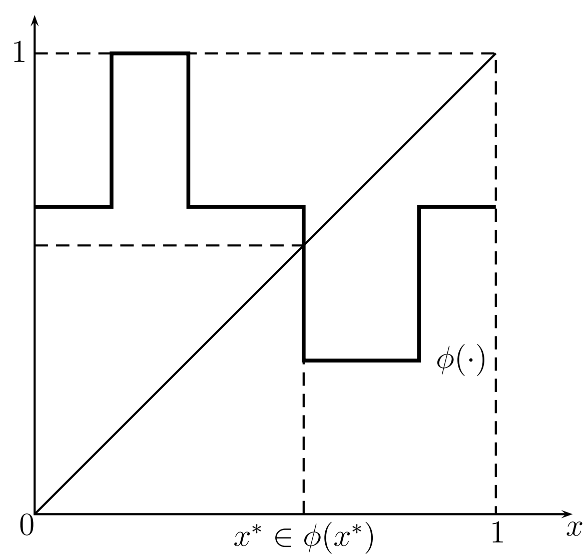
Theorem 1. There exists a unique equilibrium.

First, note that a set of stable payoff differences can be indexed by a real number $x \in [0,1]$: in Table 1, $v_{mH} - v_{mL}$ decreases from $s_{HH} - s_{LH}$ in markets 1.1 and 2.1 to $s_{Hl} - s_{Ll}$ in markets 1.7 and 2.7, so $(v_{mH} - v_{mL})_x = (1-x)(s_{HH} - s_{LH}) + x(s_{Hl} - s_{Ll})$, $x \in [0,1]$. Corresponding to every $v_{mH} - v_{mL}$ is a distinct set of $v_{wH} - v_{wL}$, $v_{wh} - v_{wl}$ and $v_{wL} - v_{wl}$. Given a set of stable payoff differences, optimal marriage timing decisions are defined as in Lemma 1; given optimal marriage timing decisions, marriage type distributions are induced as in Lemma 2; given marriage type distributions, stable matching is defined as in Lemma 3 and stable marriage payoff differences are defined as in Lemma 4. Therefore, given any set of marriage payoff differences indexed by x, we arrive at another set of marriage payoff differences indexed by x', following the composite map ϕ constructed from Lemma 1 to Lemma 4,

$$\phi : \begin{array}{c}\text{stable}\\ \text{payoff}\\ \text{differences}\\ x\end{array} \xrightarrow{\text{Lemma 1}} \begin{array}{c}\text{optimal}\\ \text{marriage}\\ \text{timing}\\ \sigma_m, \sigma_w\end{array} \xrightarrow{\text{Lemma 2}} \begin{array}{c}\text{induced}\\ \text{type}\\ \text{distributions}\\ G_m, G_w\end{array} \xrightarrow{\text{Lemma 4}} \begin{array}{c}\text{stable}\\ \text{payoff}\\ \text{differences}\\ x'\end{array}$$

If the initial set of payoff differences indexed by x coincides with the terminal set of payoff differences indexed by x', then the set forms an equilibrium. In other words, a fixed point of ϕ represents an equilibrium set of marriage payoff differences. There always exists an equilibrium: the composite map ϕ has a fixed point by Kakutani's fixed-point theorem, because x takes the value on the convex set $[0,1]$, ϕ is upper-hemicontinuous, and $\phi(x)$ is non-empty and convex for all $x \in [0,1]$. Proving equilibrium uniqueness is more complicated, because $\phi(\cdot)$ may not be monotonic (see Figure 6 for an example). Nonetheless, $\phi(\cdot)$ is monotonically decreasing around the fixed point, and the non-monotonic portion of $\phi(\cdot)$ is sufficiently away from the fixed point so $\phi(\cdot)$ always has a unique fixed point.

Figure 6: $\phi(\cdot)$ may not be monotonically decreasing but always has a unique fixed point.



4 Predictions

4.1 Men's Income by Age at Marriage

Proposition 1. The relationship between age at marriage and personal income for men is always hump-shaped in equilibrium: those who marry in the first period have an average income of y_{mL}, those who marry in the second period have an average income of y_{mH}, those who marry in the third period have an average income between y_{mL} and y_{mH}, and those who are unmarried have an average income of y_{mL}.

All ability $p < p_m^*$ men enter the marriage market in the first period with no investment and a low lifetime income. Some of them may remain unmarried. All those who get married in the first period earn a low income. Those who enter the marriage market in the second period are ability $p \geq p_m^*$ men who realize a high income from college investment, proportion p of ability $p \geq p_m^*$ men, to be exact. The rest of ability $p \geq p_m^*$ men, proportion $1 - p$ of ability $p \geq p_m^*$ men, to be exact, enter the marriage market in the third period. Among those who enter the marriage market in the third period, some of them receive a high income and some of them receive a low income. Some of the low-income career-investing men may not get married. All of those who stay unmarried have a low income.

The predicted equilibrium hump-shaped relationship is consistent with the observed relationship, and the effects generating the hump shape extend and unify previous theories.

Initial Upward-Sloping Portion. The initial upward-sloping portion of the relationship comes from the fact that higher ability men only want to marry after their labor market outcome improves and lower ability men have no incentive to delay their marriage as their income and marital prospects cannot improve substantially. The intention of delaying marriage to improve labor market outcome and thus marriage market outcome is the same as but the reason is different from Bergstrom and Bagnoli (1993). In Bergstrom and Bagnoli (1993), higher ability men wait for the second period to reveal their true abilities. Their true abilities have never changed through investments. The incentive to delay marriage comes from asymmetric information that men know their own abilities but cannot credibly signal to their partners, whereas in the current model, there is no asymmetric information, and the incentive to delay marriage comes from better payoff in the marriage market.

Latter Downward-Sloping Portion. The downward-sloping portion of the relationship cannot be explained by Bergstrom and Bagnoli (1993) although Bergstrom and Schoeni (1996) empirically establish the hump-shaped relationship. The current model exactly captures the conjectured effect in Bergstrom and Schoeni (1996),

> Some of these men who marry very late in life or not at all may be persons whose successes in life have not met the expectations that led them to postpone marriage and who continue to postpone marriage until their true worth is recognized.

The downward-sloping portion of the relationship comes from labor market friction. Men who receive a successful labor market outcome marry early, and men who fail to receive a successful labor market outcome early marry later. The prediction that conditional on education investment higher ability men tend to marry early as they are more likely to succeed earlier, is the same as the prediction in Becker (1974) and Keeley (1979), but for a different friction. In their model, higher-ability men tend to marry earlier because they encounter less marriage market friction. In this model, higher ability men tend to marry earlier because higher ability men encounter less labor market friction that translates to less marriage market friction and lower ability men involuntarily marry later because of their labor market friction that translates to marriage market friction.

Unmarried Men. Finally, if there are any unmarried men in this model, they are low-income eaners. This effect captures another remark in Bergstrom and Schoeni (1996),

> There may also be a considerable number of males who are such poor marriage material, that any female whom they would wish to marry would prefer being single to marrying one of these males.

In contrast, neither Becker (1974) nor Bergstrom and Bagnoli (1993) predict what men remain unmarried.

Marriage-Age Distribution. In addition to the relationship between marriage-age and personal income, the model predicts men's distribution of marriage-ages. In equilibrium, fraction $F_m(p_m^*)$ of men enters the marriage market in the first period, fraction $\int_{p_m^*}^1 pdF_m(p)$ of men enters the marriage market and marries in the second period, and fraction of $\int_{p_m^*}^1 (1-p)dF_m(p)$ of men enters the marriage market in the third period. Some of those who enter the marriage market in the first period and in the third period remain unmarried.

4.2 Women's Income by Age at Marriage

Proposition 2. The equilibrium relationship between women's age at marriage and personal income can be hump-shaped or positive. Those who marry in the first period have an average income of $y_{w1}^* = y_{wL}$. Those who marry in the second period have an average income of

$$y_{w2}^* = \frac{\int_{p_{w1}^*}^1 pdF_w(p)}{\int_{p_{w1}^*}^1 pdF_w(p) + \int_{p_{w1}^*}^{p_{w2}^*}(1-p)dF_w(p)} y_{wH} + \frac{\int_{p_{w1}^*}^{p_{w2}^*}(1-p)dF_w(p)}{\int_{p_{w1}^*}^1 pdF_w(p) + \int_{p_{w1}^*}^{p_{w2}^*}(1-p)dF_w(p)} y_{wL}.$$

Those who marry in the third period have an average income of

$$y_{w3}^* = \frac{\int_{p_{w2}^*}^1 (1-p)pdF_w(p)}{\int_{p_{w2}^*}^1 (1-p)[p+(1-p)r]dF_w(p)} y_{wH} + \frac{\int_{p_{w2}^*}^1 (1-p)(1-p)dF_w(p)}{\int_{p_{w2}^*}^1 (1-p)[p+(1-p)]dF_w(p)} y_{wL}.$$

Whether y_{w2}^* or y_{w3}^* is larger is indeterminate. If the probability of staying fit in the third period is sufficiently low and reproductive fitness in the marriage market is sufficiently important, then the relationship is positive, $y_{w1}^* < y_{w2}^* < y_{w3}^*$. If the probability of staying fit in the third period is sufficiently high and reproductive fitness in the marriage market is sufficiently less important, then the relationship is hump-shaped, $y_{w1}^* < y_{w3}^* < y_{w2}^*$.

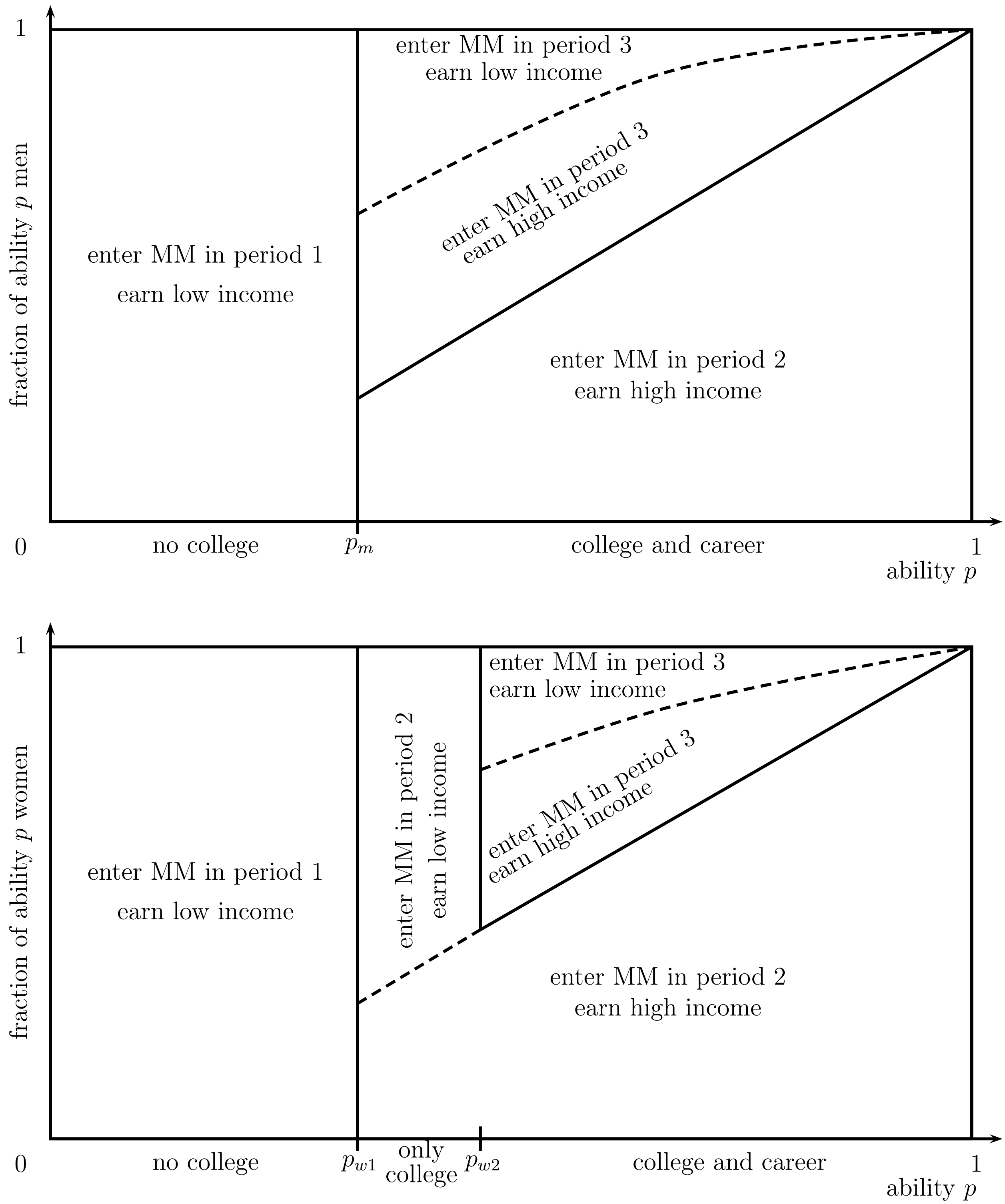


Figure 7: Optimal investment and marriage timing as well as labor-market outcome.

The bottom panel of Figure 7 illustrates women's equilibrium marriage timing and their equilibrium labor-market and marriage-market outcomes, in contrast to the top panel of Figure 7 that illustrates men's. All ability $p < p^*_{w1}$ women do not go to college, enter the marriage market in the first period, and receive a low income in the labor market. Ability $p^*_{w1} \leq p < p^*_{w2}$ women enter the marriage market in the second period regardless of the outcome of their first period investments. Some of them, proportion p of ability p women to be exact, earn a high income and the rest of them earn a low income. Although they differ in their income dimension, all of them are fit when they enter the marriage market in the second period. Among ability $p \geq p^*_{w2}$ women, proportion p of ability p women enter the marriage market in the second period and earn a high income. The rest marry in the third period, and they have either high income or low income and become fit or less fit, depending on the realizations of their income and fitness shocks.

Unlike that men who marry in the second period all have high income, women who marry in the second period consist of two income groups: those who have ability between p^*_{w1} and p^*_{w2} and fail to receive a high-income offer right out of college, and those who have ability above p^*_{w1} and successfully receive a high-income offer in the second period. The mass of the low-income group is $\int_{p^*_{w1}}^{p^*_{w2}}(1-p)dF_w(p)$, and the mass of the high-income group is $\int_{p^*_{w1}}^{1} pdF_w(p)$. The average income of women who marry in period 2 is

$$y^*_{w2} = \frac{\int_{p^*_{w1}}^{1} pdF_w(p)}{\int_{p^*_{w1}}^{1} pdF_w(p) + \int_{p^*_{w1}}^{p^*_{w2}}(1-p)dF_w(p)} y_{wH} + \frac{\int_{p^*_{w1}}^{p^*_{w2}}(1-p)dF_w(p)}{\int_{p^*_{w1}}^{1} pdF_w(p) + \int_{p^*_{w1}}^{p^*_{w2}}(1-p)dF_w(p)} y_{wL}.$$

In comparison, those who enter the marriage market in the third period are ability $p \geq p^*_{w2}$ women who fail to receive a high income in the second period. Among these, proportion p of ability p women receive a high income and proportion $1-p$ of ability p women receive a low income. The average income of women who marry in period 3 is

$$y^*_{w3} = \frac{\int_{p^*_{w2}}^{1}(1-p)pdF_w(p)}{\int_{p^*_{w2}}^{1}(1-p)dF_w(p)} y_{wH} + \frac{\int_{p^*_{w2}}^{1}(1-p)(1-p)dF_w(p)}{\int_{p^*_{w2}}^{1}(1-p)dF_w(p)} y_{wL}.$$

Initial Upward-Sloping Portion. The average income of the women who marry in period 1, y^*_{w1}, is the lowest. Although Becker (1973) also predicts an upward-sloping portion of the relationship for women who marry between thirty, the same result follows a different reason in this paper. In this paper, the initial upward-sloping portion of the relationship follows from an investment effect. Those delay their marriages initially tend to engage in human capital investments that improve their income and marriage prospects. This effect is completely missing in Becker (1973) and Bergstrom and Bagnoli (1993) because they do not consider the possibility of women improving their labor-market outcomes, making their theories inadequate to the changing society where females are gradually achieving social and economic equalities.

Change in the Latter Portion: Upward-Sloping to Downward-Sloping. While y^*_{w1} is the lowest, whether y^*_{w2} or y^*_{w3} is higher depends on the parameters of the model. If p^*_{w2} is small and very close to p^*_{w1}, then most of the women who marry in the second period have a high income, and the average income of those who marry in the second period is close to y_{wH} and is larger than the average income of those who marry in the third period. In contrast, if p^*_{w2} is large and close to 1, then a significant portion of women who marry in the second period have a low income, and the average income of those who marry in the second period is lower than those who marry in the third period, as those who marry in the third period consist mostly of high-ability women who are unlucky in the second period and will eventually achieve a high income in the third period.

Whether p^*_{w2} is large or small depends on the parameters of the model. Recall

$$p^*_{w2} = \frac{c_w + (1-r)(v^*_{wL} - v^*_{wl})}{y_{wH} - y_{wL} + r(v^*_{wH} - v^*_{wL}) + (1-r)(v^*_{wh} - v^*_{wl})}.$$

Holding everything else constant, p^*_{w2} is smaller when r is larger, $y_{mH}-y_{mL}$ is larger, $v^*_{wH}-v^*_{wL}$ is larger, $v^*_{wh} - v^*_{wl}$ is larger, and/or $v^*_{wL} - v^*_{wl}$ is smaller.

Two factors lower the threshold ability p^*_{w2}. First, when women's probability of staying fit in the third period increases, more women incline to delay marriage because they face a lower chance of decreasing their fitness and the associated marriage payoff in the marriage market. Second, when the relative importance of reproductive fitness in the marriage market declines, manifested in a smaller decline in less fit women's marriage payoff $v^*_{wL} - v^*_{wl}$ and/or higher gain in women's labor market and marriage market associated with a higher income, i.e. increases in $y_{wH} - y_{wL}$, $v^*_{wH} - v^*_{wL}$, and $v^*_{wh} - v^*_{wl}$. When women stay fit with a higher probability and/or the relative importance of reproductive fitness decreases, y^*_{w2} becomes larger and y^*_{w3} becomes smaller, and the equilibrium relationship becomes hump-shaped. In the extreme case when $r = 1$ or the reproductive fitness is ignored in the marriage market, women's equilibrium strategies are the same as men's, $p^*_{w1} = p^*_{w2}$, all the women who marry in the second period earn a high income.

Changes in the two contributing factors can be thought of as changes in the supply and demand of reproductive capital in the marriage market, respectively. The increase in women's probability of staying fit can be treated as an increase in women's supply of reproductive fitness, whereas the decline in the relative importance and the associated relative value of reproductive fitness is a decrease in the demand for reproductive capital by men in the marriage market. These two factors are consistent with empirical evidence. On one hand, the increase in the supply of reproductive fitness has been achieved by advances in medical technology such as in-vitro fertilization, egg-freezing, and better maternal health services resulting in higher probability of staying fit and conceiving. Older women can have children with more ease and less adverse health effects. On the other hand, the decline in the demand for reproductive fitness is a result of several social changes. First, the desired family size has decreased; in the United States, the average desired number of children has declined from 3.6 to 2.6 from 1960 to 2010. Many families have shifted from demand for quantity of children to demand for quality (Becker and Lewis, 1973). Women's reproductive fitness becomes less of a concern than women's income and education in marriage decisions. Second, an increase in income gain from college and career investments also contributes to a decrease in the relative importance of reproductive fitness. When income gain becomes more important, gender difference in the marriage market becomes less important and the relationships between age at marriage and income become similar for the two genders.

Unmarried Women. Less fit low-income women are the most likely to be unmarried, followed by less fit high-income women when reproductive fitness is more important than income in the marriage market, or by fit low-income women when reproductive fitness is less important than income. Fit high-income women are the most likely to be married. This prediction is consistent with the finding that educated women have higher marriage rates than uneducated women.

Marriage-Age Distribution. The mass of women marrying in period 1 is $F_w(p^*_{w1})$, the mass of women marrying in period 2 is $\int_{p^*_{w1}}^{1} p dF_w(p) + \int_{p^*_{w1}}^{p^*_{w2}} (1-p)dF_w(p)$, and the mass of women marrying in period 3 is $\int_{p^*_{w2}}^{1} (1-p)dF_w(p)$.

4.3 Husband's Income by Wife's Age at Marriage

Proposition 3. Consider the relationship between wife's age at marriage and husband's income in equilibrium.

a. The relationship is always hump-shaped: those who marry in the second period have the highest-income husbands on average, and those who marry in the first period or in the third period have lower-income husbands on average.

b. The relationship can be right-skewed or left-skewed.

   i. When reproductive fitness is relatively important in the marriage market and the probability of fitness is low, the relationship is right-skewed: the women who marry in period 1 have higher-income husbands on average than those who marry in period 3.

   ii. When reproductive fitness is relatively less important in the marriage market and the probability of fitness is high, the relationship is left-skewed: the women who marry in period 1 have lower-income husbands on average than those who marry in period 3.

The proposition hinges on the different stable matching patterns depending on the relative importance of reproductive fitness, in particular Lemma 3c. Recall that, when fitness is more important than income in the marriage market, $s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$, a fit low-income woman almost always marries a higher-income husband than a less fit high-income woman (markets 2.1-2.7 in Figure 5), and in contrast, when fitness is less important than income in the marriage market, i.e. $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$, a fit low-income woman almost always marries a lower-income husband than a less fit high-income woman (markets 1.1-1.7 in Figure 5). Let $x^*_{w\tau_w}$ denote the average income of the men who marry type τ_w women. By Lemma 3a, $x^*_{wH} \geq x^*_{wL}$ and $x^*_{wh} \geq x^*_{wl}$ and by Lemma 3b, $x^*_{wH} \geq x^*_{wh}$ and $x^*_{wL} \geq x^*_{wl}$. By Lemma 3c, $x^*_{wh} \leq x^*_{wL}$ when $s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$, and $x^*_{wh} \geq x^*_{wL}$ when $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$.

Initial Upward-Sloping Portion. The upward-sloping portion of the hump-shaped relationship in Proposition 3a is straightforward to see. The women who marry in period 1 are reproductively fit low-income earners. Let x^*_{wa} denote the average income of the men who marry women who marry in period a. Then $x^*_{w1} = x^*_{wL}$. The women who marry in period 2 are a mix of reproductively fit low-income earners and reproductively fit high-income earners,

$$x^*_{w2} = \frac{\int_{p^*_{w1}}^{1} pdF_w(p)}{\int_{p^*_{w1}}^{1} pdF_w(p) + \int_{p^*_{w1}}^{p^*_{w2}}(1-p)dF_w(p)} x^*_{wH} + \frac{\int_{p^*_{w1}}^{p^*_{w2}}(1-p)dF_w(p)}{\int_{p^*_{w1}}^{1} pdF_w(p) + \int_{p^*_{w1}}^{p^*_{w2}}(1-p)dF_w(p)} x^*_{wL}.$$

Since reproductively fit high-income women always marry higher-income husbands than reproductively fit low-income women do ($x^*_{wH} \geq x^*_{wL}$), on average the women who marry in period 2 always have higher-income husbands than those who marry in period 1.

Latter Downward-Sloping Portion. In comparison, the downward-sloping portion of the hump-shaped relationship and the different skewnesses of the relationship are more complicated and intricate. The women who marry in period 3 are mixed of all marriage types,

$$x^*_{w3} = \frac{\int_{p^*_{w2}}^{1}(1-p)pdF_w(p)}{\int_{p^*_{w2}}^{1}(1-p)dF_w(p)}[rx^*_{wH} + (1-r)x^*_{wh}] + \frac{\int_{p^*_{w2}}^{1}(1-p)(1-p)dF_w(p)}{\int_{p^*_{w2}}^{1}(1-p)dF_w(p)}[rx^*_{wL} + (1-r)x^*_{wl}].$$

When the reproductive fitness is relatively important and the probability of fitness is low, the average personal income of the women who marry in period 3 is higher than that of those who marry in period 2, as Proposition 2 shows. However, many of those who marry in period 3 become less fit so they become type h. Compared to most of the type L women in period 2, those women who marry in period 3 are mostly of type h and l and consequently marry lower-income husbands. In contrast, when reproductive fitness is relatively less important, men and women are assortatively matched on incomes. Coupled with the fact that the probability of staying fit is large, the women who marry in period 3 have on average lower personal income than those who marry in period 2, as Proposition 2 shows. Since men and women are assortatively matched on incomes, the women who marry in period 3 and have lower average personal income on average have a lower-income husband than those who marry in period 2 and have higher average personal income.

Change in Matching Patterns. Proposition 3b follows from the same logic explaining the downward-sloping portion of the hump-shaped relationship in Proposition 3a. The comparison between the women who marry in period 1 and those who marry in period is mostly a comparison between the marital prospects of type L women and type h women. When the reproductive fitness is more important, type L women have better marital prospects measured in terms of spousal income, hence the average income of the husbands of the women who marry in period 3 is higher. When the reproductive fitness is relatively less important, type h women have higher better marital prospects than type L women, and coupled with the condition that the probability of staying fit is high so that many of the women who marry in period 3 stay to be type H women instead of type h women, those who marry in period 3 have a higher average spousal income than those who marry in period 1.

Empirically, the same factors that resulted in the change from a positive relationship between age at marriage and personal income for women to a hump-shaped relationship also contributed to the change of the relationship from right-skewed to left-skewed.

5 A Numerical Example

In this section, I show numerically how the aforementioned changes in the supply and demand of reproductive fitness result in the empirically observed changes in the relationships. Since all of the equilibrium components can be characterized in closed forms, we can numerically demonstrate the theoretical predictions of the model given any set of parameters. In particular, I illustrate all of the previous theoretical arguments using a numerical example that compares two regimes: (1) *Old Regime*: reproductive fitness is relatively more important in the marriage market and the probability of staying fit is low and (2) *New Regime*: reproductive fitness is relatively less important in the marriage market and the probability of staying fit is high.

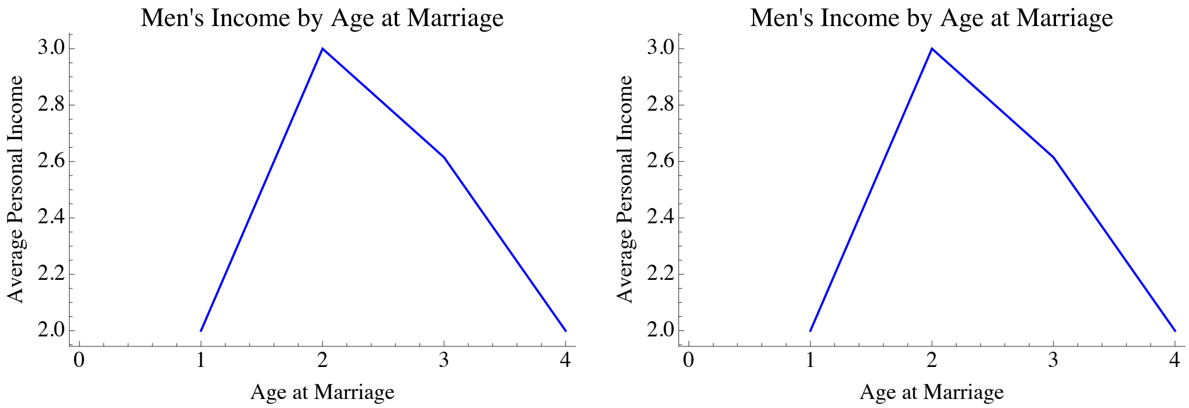
I compare the equilibrium relationships in two regimes. Table 2 shows the parameters in the two regimes. The only differences are in reproductive fitness probability r and in marriage surplus s. In the old regime, the probability of staying fit is 0.6, whereas in the new regime, the probability of staying increases to 0.8. The relative importance of reproductive fitness is bigger in the old regime than in the new regime, as the drop in a couple's marriage surplus is bigger if the wife is less fit. I keep all parameters constant except for the fitness-related parameters to highlight the importance of these parameters.[7]

	Old Regime	New Regime
$F_m(p)$	p^2	p^2
$F_w(p)$	p^2	p^2
c_m	0.5	0.5
c_w	0.5	0.5
y_{mH}	3.0	3.0
y_{mL}	2.0	2.0
y_{wH}	2.1	2.1
y_{wL}	1.4	1.4
r	0.6	0.8
$s_{\tau_m \tau_w}$ if fit	$\sqrt{y_{m\tau_m} y_{w\tau_w}}$	$\sqrt{y_{m\tau_m} y_{w\tau_w}}$
$s_{\tau_m \tau_w}$ if less fit	$\sqrt{0.3 y_{m\tau_m} y_{w\tau_w}}$	$\sqrt{0.4 y_{m\tau_m} y_{w\tau_w}}$

Table 2: Parameters used in the numerical example: the two regimes only differ in reproductive fitness probability r and marriage surplus s.

The predicted relationships are qualitatively consistent with the empirical observations. Figure 8 shows the relationships between men's age at marriage and their personal income. The relationship stays to be hump-shaped in both regimes. Figure 9 shows the relationships between women's age at marriage and their personal income. The relationship is positive in the old regime and is hump-shaped in the new regime. Figure 10 shows the relationships between wife's age at marriage and husband income. In both regimes, the relationship

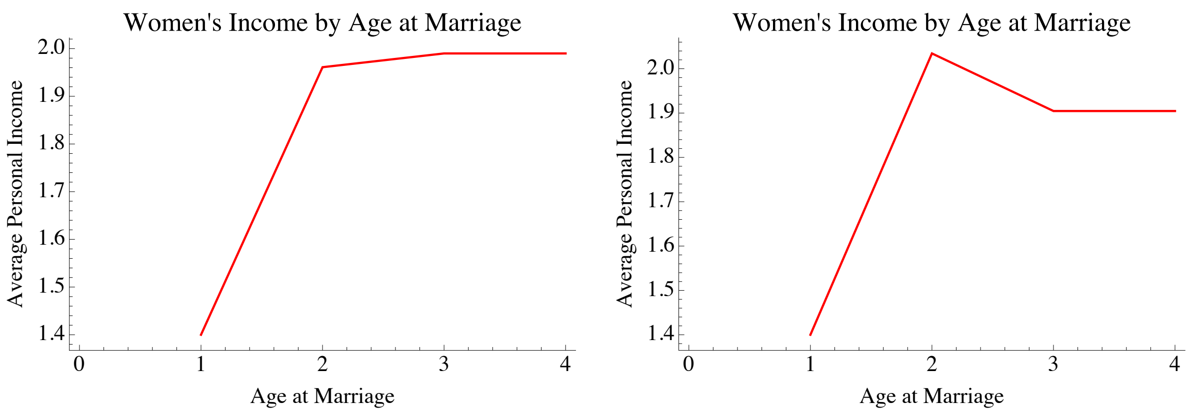
[7] The Mathematica codes are available at http://www.hanzhezhang.net/research/1606MarriageAge_Mathematica.nb and can be easily manipulated.



$F_m(p)=p^2$, $F_w(p)=p^2$, $r=0.6$, $c_m=0.5$, $c_w=0.5$, $y_{mH}=3.0$, $y_{mL}=2.0$, $y_{wH}=2.1$, $y_{wL}=1.4$, $s_{HH}=2.5$, $s_{HL}=2.0$, $s_{LH}=2.0$, $s_{LL}=1.7$, $s_{Hh}=1.4$, $s_{Hl}=1.1$, $s_{Lh}=1.1$, $s_{Ll}=0.9$.

$F_m(p)=p^2$, $F_w(p)=p^2$, $r=0.8$, $c_m=0.5$, $c_w=0.5$, $y_{mH}=3.0$, $y_{mL}=2.0$, $y_{wH}=2.1$, $y_{wL}=1.4$, $s_{HH}=2.5$, $s_{HL}=2.0$, $s_{LH}=2.0$, $s_{LL}=1.7$, $s_{Hh}=1.6$, $s_{Hl}=1.3$, $s_{Lh}=1.3$, $s_{Ll}=1.1$.

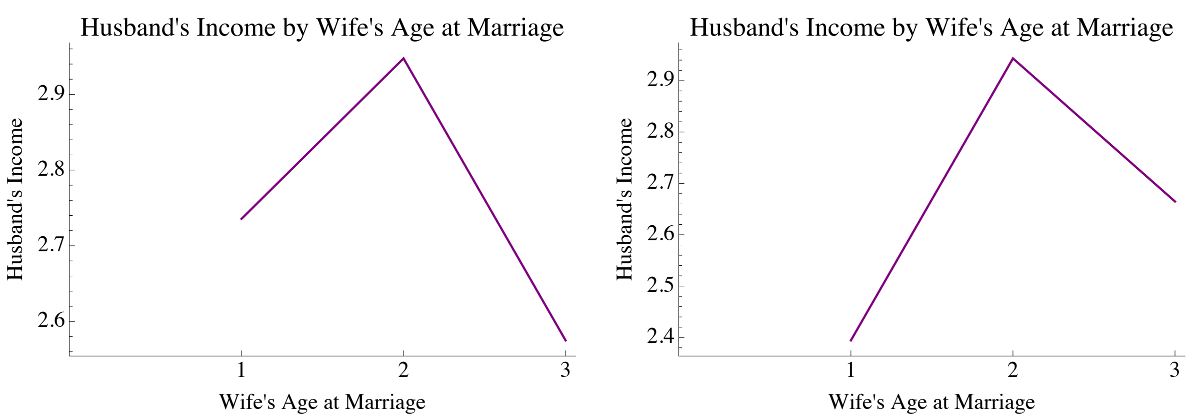
Figure 8: Men's Income by Age at Marriage in Equilibrium.



$F_m(p)=p^2$, $F_w(p)=p^2$, $r=0.6$, $c_m=0.5$, $c_w=0.5$, $y_{mH}=3.0$, $y_{mL}=2.0$, $y_{wH}=2.1$, $y_{wL}=1.4$, $s_{HH}=2.5$, $s_{HL}=2.0$, $s_{LH}=2.0$, $s_{LL}=1.7$, $s_{Hh}=1.4$, $s_{Hl}=1.1$, $s_{Lh}=1.1$, $s_{Ll}=0.9$.

$F_m(p)=p^2$, $F_w(p)=p^2$, $r=0.8$, $c_m=0.5$, $c_w=0.5$, $y_{mH}=3.0$, $y_{mL}=2.0$, $y_{wH}=2.1$, $y_{wL}=1.4$, $s_{HH}=2.5$, $s_{HL}=2.0$, $s_{LH}=2.0$, $s_{LL}=1.7$, $s_{Hh}=1.6$, $s_{Hl}=1.3$, $s_{Lh}=1.3$, $s_{Ll}=1.1$.

Figure 9: Women's Income by Age at Marriage in Equilibrium.



$F_m(p)=p^2$, $F_w(p)=p^2$, $r=0.6$, $c_m=0.5$, $c_w=0.5$, $y_{mH}=3.0$, $y_{mL}=2.0$, $y_{wH}=2.1$, $y_{wL}=1.4$, $s_{HH}=2.5$, $s_{HL}=2.0$, $s_{LH}=2.0$, $s_{LL}=1.7$, $s_{Hh}=1.4$, $s_{Hl}=1.1$, $s_{Lh}=1.1$, $s_{Ll}=0.9$.

$F_m(p)=p^2$, $F_w(p)=p^2$, $r=0.8$, $c_m=0.5$, $c_w=0.5$, $y_{mH}=3.0$, $y_{mL}=2.0$, $y_{wH}=2.1$, $y_{wL}=1.4$, $s_{HH}=2.5$, $s_{HL}=2.0$, $s_{LH}=2.0$, $s_{LL}=1.7$, $s_{Hh}=1.6$, $s_{Hl}=1.3$, $s_{Lh}=1.3$, $s_{Ll}=1.1$.

Figure 10: Husband's Income by Wife's Age at Marriage in Equilibrium.

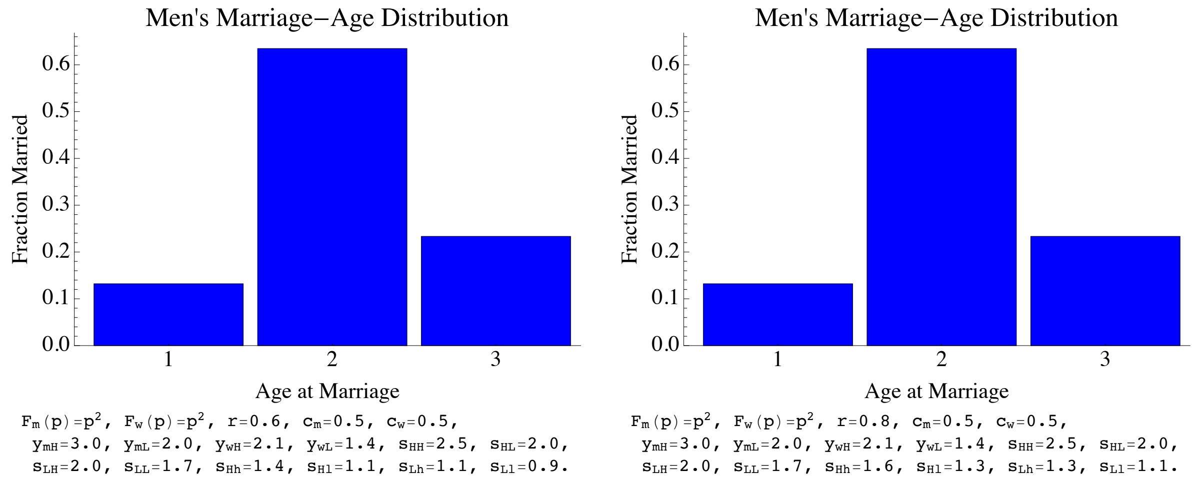


Figure 11: Men's Marriage-Age Distribution in Equilibrium.

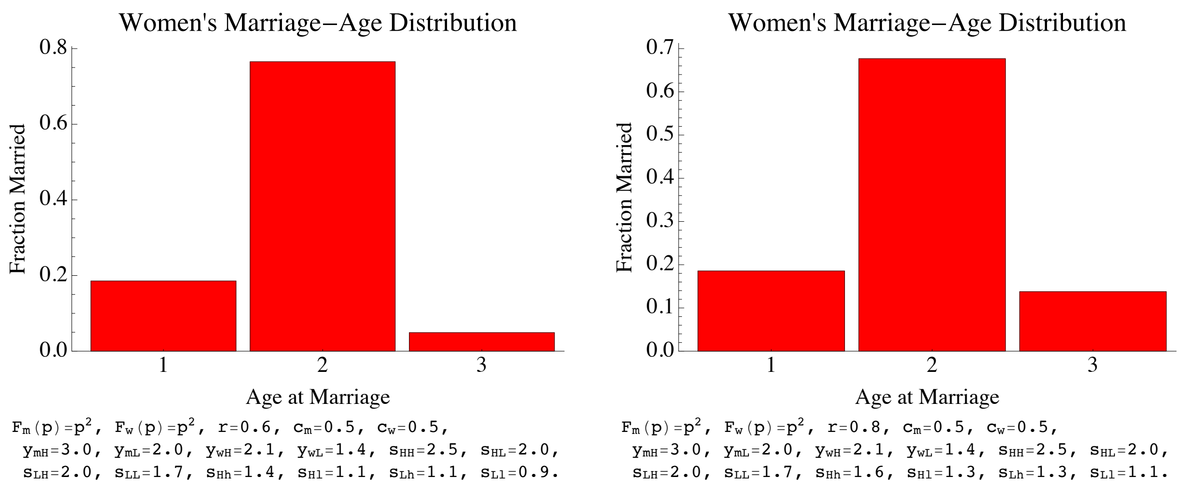


Figure 12: Women's Marriage-Age Distribution in Equilibrium.

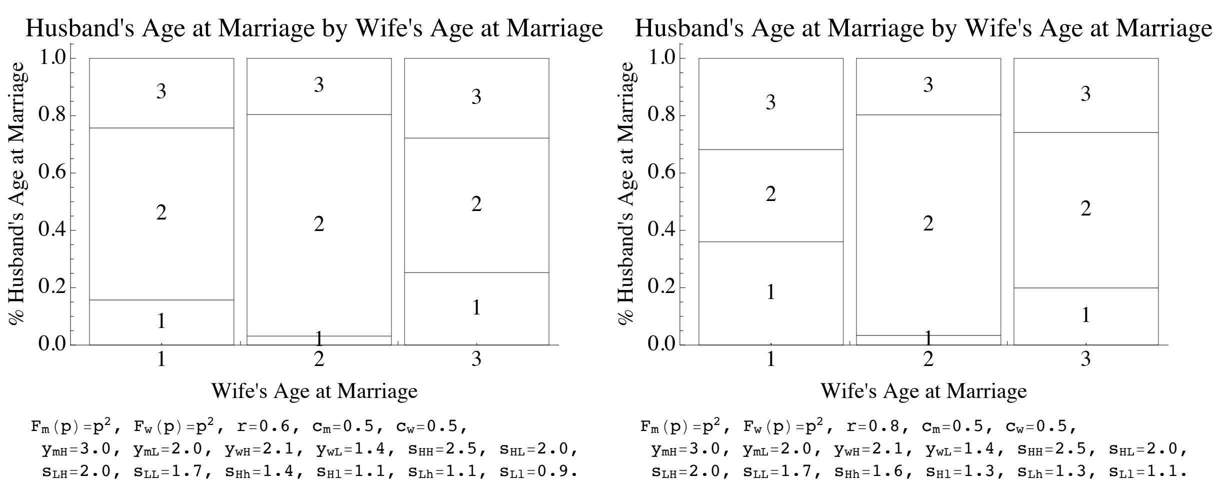


Figure 13: Wife's Age at Marriage by Husband's Age at Marriage in Equilibrium.

is hump-shaped. In addition, in the old regime, the relationship is right-skewed: those who marry earlier marry higher-income husbands on average, and in the new regime, the relationship is left-skewed: those who marry earlier marry lower-income husbands on average.

In addition to the relationships that qualitatively match the observed patterns, the model also predicts marriage-age distributions, as shown in Figures 11 and 12. Comparing between men and women, men tend to marry later than women, as fewer men marry in the first period and more men marry in the second period as well as in the third period. Comparing the new regime and the old regime, due to the change in both supply and demand of reproductive fitness factors, more women tend to marry later.

More interestingly, the model generates patterns regarding matching by ages, as shown in Figure 13. Although the preferences for age are not directly assumed, because age at marriage in equilibrium is correlated with reproductive fitness level and lifetime income, there are still non-trivial distributions of couples of different age pairs. In general, men marry later than women, and (as a result) men tend to marry younger women. There is a degree of assortative matching by ages, as majority of age-2 and age-3 men marry age-2 women and majority of age-1 men marry age-1 women. The result is not due to assortative matching directly on ages, but due to partially assortative matching in incomes and fitnesses which are correlated with ages. Because of the uncertainties in incomes and fitnesses, there is no pure assortative matching by age, as incomes and fitness differ across different individuals with the same age at marriage. These theoretical predictions on matching by ages can be tested against data if such data is available.

6 Conclusion

This paper documents the relations of age at marriage with American men and women's labor-market outcomes and marriage-market outcomes, and the changes in these relations over the twentieth century. Motivated by these relationships and changes only partially explained by previous theories, an general-equilibrium investment-and-matching model based on human capital investments and parsimoniously gender difference in reproductive length explains the documented relationships. The model not only explains these relationships between age at marriage and socioeconomic outcomes, but also offers predictions about marriage-age distributions and matching by marriage-ages. The model is also consistent with other basic facts explained by previous literature, including but not limited to the effects of education on marriage-ages and marriage rates.

It is worth emphasizing that the paper does not intend to quantify all factors that contribute to marriage timing decisions, but rather intends to emphasize the parallel importance of human capital investments, differential fecundity, and the interaction of the two factors to other factors such as search frictions and labor specialization in the household emphasized in Becker (1973) and asymmetric information emphasized in Bergstrom and Bagnoli (1993), in explaining labor-market and marriage-market patterns. However, due to the tractability and flexibility of the model, it can potentially explain more labor-market and marriage-market phenomena unexplainable by previous theories.

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Appendix

Proof of Lemma 3

All three components of the lemma take the following form: for $\tau_w, \tau'_w \in \{H, L, h, l\}$, when $s_{H\tau_w} + s_{L\tau'_w} > s_{H\tau'_w} + s_{L\tau_w}$, a τ_w woman almost always marries a higher-income husband than a τ'_w woman. It suffices to show that for $\tau_w, \tau'_w \in \{H, L, h, l\}$, when $s_{H\tau_w} + s_{L\tau'_w} > s_{H\tau'_w} + s_{L\tau_w}$, there is zero mass of (H, τ'_w) or (L, τ_w) couples. Suppose by way of contradiction positive masses of both (H, τ'_w) and (L, τ_w) couples. By the stability condition that couples divide up their surpluses, $v_{mH} + v_{w\tau'_w} = s_{H\tau'_w}$ and $v_{mL} + v_{w\tau_w} = s_{L\tau_w}$. In addition, by the stability condition that there is no blocking pair, $v_{mH} + v_{w\tau_w} \geq s_{H\tau_w}$ and $v_{mL} + v_{w\tau'_w} \geq s_{L\tau'_w}$. The four stability conditions together yield $s_{H\tau'_w} + s_{L\tau_w} \geq s_{H\tau_w} + s_{L\tau'_w}$, which contradicts the assumption $s_{H\tau'_w} + s_{L\tau_w} < s_{H\tau_w} + s_{L\tau'_w}$. □

Proof of Theorem 1

Preliminary Definitions. First, define the stable marriage payoff difference

$$(v_{mH} - v_{mL})_x \equiv (1 - x)(s_{HH} - s_{LH}) + x(s_{Hl} - s_{Ll}).$$

From Table 1, $v_{mH} - v_{mL} = (v_{mH} - v_{mL})_x$ where

$$x \begin{cases} = 0, & \text{Market 1.1} \\ \in [0, x_{1.3}], & \text{Market 1.2} \\ = x_{1.3}, & \text{Market 1.3} \\ \in [x_{1.3}, x_{1.5}], & \text{Market 1.4} \\ = x_{1.5}, & \text{Market 1.5} \\ \in [x_{1.5}, 1], & \text{Market 1.6} \\ = 1, & \text{Market 1.7} \end{cases}, \quad x \begin{cases} = 0, & \text{Market 2.1} \\ \in [0, x_{2.3}], & \text{Market 2.2} \\ = x_{2.3}, & \text{Market 2.3} \\ \in [x_{2.3}, x_{2.5}], & \text{Market 2.4} \\ = x_{2.5}, & \text{Market 2.5} \\ \in [x_{2.5}, 1], & \text{Market 2.6} \\ = 1, & \text{Market 2.7} \end{cases},$$

in which $x_{1.3}$, $x_{2.5}$, $x_{1.5}$, and $x_{2.3}$ are defined as the solutions to $(v_{mH} - v_{mL})_{x_{1.3}} = (v_{mH} - v_{mL})_{x_{2.5}} = s_{Hh} - s_{Lh}$, and $(v_{mH} - v_{mL})_{x_{1.5}} = (v_{mH} - v_{mL})_{x_{2.3}} = s_{HL} - s_{LL}$. Given $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$ (Markets 1.1-1.7) or $s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$ (Markets 2.1-2.7), each different set of stable marriage payoff differences has a unique $(v_{mH} - v_{mL})_x$ where $x \in [0, 1]$. Let $(v_{wH} - v_{wL})_x$, $(v_{wh} - v_{wl})_x$, and $(v_{wL} - v_{wl})_x$ represent the stable marriage payoff differences accompanying $(v_{mH} - v_{mL})_x$.

Second, define optimal cutoffs given stable marriage payoff differences indexed by x,

$$p_m(x) \equiv c_m/[y_{mH} - y_{mL} + (v_{mH} - v_{mL})_x],$$

$$p_{w1}(x) \equiv c_w/[y_{wH} - y_{wL} + (v_{wH} - v_{wL})_x],$$

$$p_{w2}(x) \equiv \frac{c_w + (1 - r)(v_{wL} - v_{wl})_x}{y_{wH} - y_{wL} + r(v_{wH} - v_{wL})_x + (1 - r)(v_{wh} - v_{wl})_x} < 1.$$

Third and finally, define the optimally induced marriage-market masses given the stable payoff differences indexed by x,

$$G_{mH}(x) \equiv \int_{p_m(x)}^{1} p_m(2-p_m)dF_m(p_m),$$

$$G_{wH}(x) \equiv \int_{p_{w1}(x)}^{1} pdF_w(p) + \int_{p_{w2}(x)}^{1} r(1-p)pdF_w(p),$$

$$G_{wh}(x) \equiv \int_{p_{w2}(x)}^{1} (1-r)(1-p)pdF_w(p),$$

$$G_{wL}(x) \equiv F_w(p_{w1}(x)) + \int_{p_{w1}(x)}^{p_{w2}(x)} (1-p)dF_w(p) + \int_{p_{w2}(x)}^{1} r(1-p)^2 dF_w(p),$$

and the sums of the masses are

$$G_{wH}(x) + G_{wh}(x) = \int_{p_{w1}(x)}^{1} pdF_w(p) + \int_{p_{w2}(x)}^{1} (1-p)pdF_w(p),$$

$$G_{wH}(x) + G_{wL}(x) = F_w(p_{w2}(x)) + \int_{p_{w2}(x)}^{1} [p + r(1-p)]dF_w(p),$$

$$G_{wH}(x) + G_{wh}(x) + G_{wL}(x) = F_w(1) - \int_{p_{w2}(x)}^{1} (1-r)(1-p)dF_w(p).$$

We subsequently prove the theorem under two cases: (Case 1.) $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$ and (Case 2.) $s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$.

Case 1. First, suppose $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$. Define $\phi_1(\cdot)$ to be the correspondence that maps an index x of stable marriage payoff differences to an index or a set of indices of stable marriage payoff differences in the marriage market induced by the optimal marriage timing when players expect the stable marriage payoff differences indexed by x,

$$\phi_1(x) \equiv \begin{cases} \{0\}, & G_{mH}(x) \in [0, G_{wH}(x)) \\ [0, x_{1.3}], & G_{mH}(x) = G_{wH}(x) \\ \{x_{1.3}\}, & G_{mH}(x) \in (G_{wH}(x), G_{wH}(x) + G_{wh}(x)) \\ [x_{1.3}, x_{1.5}], & G_{mH}(x) = G_{wH}(x) + G_{wh}(x) \\ \{x_{1.5}\}, & G_{mH}(x) \in (G_{wH}(x) + G_{wh}(x), G_{wH}(x) + G_{wh}(x) + G_{wL}(x)) \\ [x_{1.5}, 1], & G_{mH}(x) = G_{wH}(x) + G_{wh}(x) + G_{wL}(x) \\ \{1\}, & G_{mH}(x) \in (G_{wH}(x) + G_{wh}(x) + G_{wL}(x), 1] \end{cases}$$

Figures A1 and A2 provide two examples illustrating how $\phi_1(\cdot)$ is constructed.

Claim 1A. $\phi_1(\cdot)$ has a fixed point $x^* \in \phi_1(x^*)$.

Figure A1: An illustration of constructing $\phi_1(\cdot)$ and finding the unique fixed point $x^* \in \phi_1(x^*)$: an example where ϕ_1 is monotonically decreasing and has a unique fixed point.

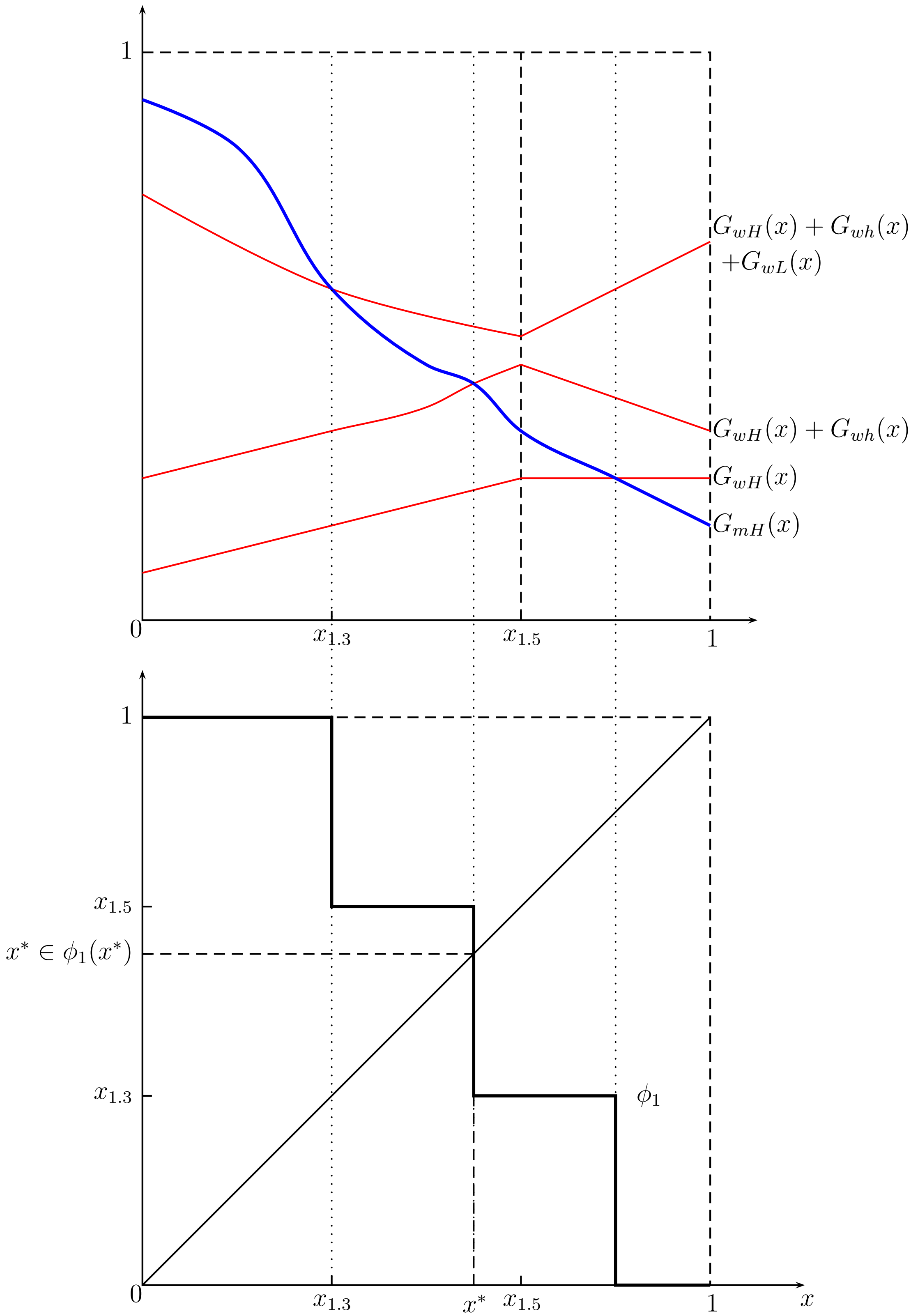


Table 3: Directions of changes in marriage payoff differences and marriage-market masses as x increases: $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$.

	$s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$		
	$0 \le x \le x_{1.3}$	$x_{1.3} \le x \le x_{1.5}$	$x_{1.5} \le x \le 1$
$(v_{mH} - v_{mL})_x$	$\downarrow$	$\downarrow$	$\downarrow$
$(v_{wH} - v_{wL})_x$	$\uparrow$	$\uparrow$	-
$(v_{wh} - v_{wl})_x$	-	$\uparrow$	$\uparrow$
$(v_{wL} - v_{wl})_x$	-	-	$\uparrow$
$p_m(x)$	$\uparrow$	$\uparrow$	$\uparrow$
$p_{w1}(x)$	$\downarrow$	$\downarrow$	-
$p_{w2}(x)$	$\downarrow$	$\downarrow$	$\uparrow^*$
$G_{mH}(x)$	$\downarrow$	$\downarrow$	$\downarrow$
$G_{wH}(x)$	$\uparrow$	$\uparrow$	-
$G_{wH}(x) + G_{wh}(x)$	$\uparrow$	$\uparrow$	$\downarrow$
$G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$	$\downarrow$	$\downarrow$	$\uparrow$

Proof of Claim 1A. $\phi_1(\cdot)$ is upper-hemicontinuous because $G_{mH}(x)$, $G_{wH}(x)$, $G_{wh}(x)$, and $G_{wL}(x)$ are continuous as $p_m(x)$, $p_{w1}(x)$, and $p_{w2}(x)$ are continuous. $\phi_1(x)$ is non-empty and convex for all x. Therefore, by Kakutani's fixed point theorem, $\phi_1 : [0,1] \twoheadrightarrow [0,1]$ has a fixed point x^* such that $x^* \in \phi_1(x^*)$. □

Claim 1B. $\phi_1(\cdot)$ has a unique fixed point $x^* \in \phi_1(x^*)$.

Proof of Claim 1B. If $\phi_1(\cdot)$ is monotonically decreasing, then uniqueness of the fixed point easily follows (an example shown by Figure A1). Unfortunately, $\phi_1(\cdot)$ is not necessarily monotonically decreasing, because $G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$ and $G_{wH}(x) + G_{wh}(x)$ are not monotonic (shown by Figure A2), so we need more work to show that there is always a unique fixed point. Fortunately, the values of x where non-monotonicity occurs can be shown not to be fixed points, case by case, as follows. In essence, the following cases can be summarized by the following statement: ϕ_1 is locally monotonic around the fixed point, and the non-monotonic portion is bounded away from the 45-degree line.

Summarized in Table 3 are the directions of changes in the marriage payoff differences, probability cutoffs, and marriage-market masses needed in the different steps of the proof. Whereas the other signs can be derived directly from definitions, the * sign, i.e. the direction of change of p_{w2} with respect to x when $x_{1.5} \le x \le 1$, requires some steps in the derivation:

$$\begin{aligned}
p_{w2}(x) &= \frac{c_w + (1-r)(v_{wL} - v_{wl})_x}{y_{wH} - y_{wL} + r(v_{wh} - v_{wl})_x + (1-r)(v_{wh} - v_{wl})_x} \\
&= \frac{c_w + (1-r)[(1-\lambda(x))(s_{LL} - s_{Ll}) + \lambda(x)(s_{HL} - s_{Hl})]}{k(x) + (1-r)[(1-\lambda(x))(s_{Hh} - s_{HL} + s_{LL} - s_{Ll}) + \lambda(x)(s_{Hh} - s_{Hl})]} \\
&= \frac{c_w + (1-r)[s_{LL} - s_{Ll} + \lambda(x)(s_{HL} - s_{Hl} - s_{LL} + s_{Ll})]}{k(x) + (1-r)[s_{Hh} - s_{HL} + s_{LL} - s_{Ll} + \lambda(x)(s_{HL} - s_{Hl} - s_{LL} + s_{Ll})}
\end{aligned}$$

where $k(x) = y_{wH} - y_{wL} + r(s_{HH} - s_{HL})$ and $\lambda(x) = (x - x_{1.5})/(1 - x_{1.5})$. Then

$$p_{w2}(x) = \frac{\text{constant}_1 + (1-r)\lambda(x)(s_{HL} - s_{Hl} - s_{LL} + s_{Ll})}{\text{constant}_2 + (1-r)\lambda(x)(s_{HL} - s_{Hl} - s_{LL} + s_{Ll})} < 1$$

strictly increases in x on $[x_{1.5}, 1]$.

1. Suppose that there is a fixed point $x^* < x_{1.5}$ (Figures A1 and A2 are both examples illustrating this case). Then $G_{mH}(x^*) < G_{wH}(x^*) + G_{wh}(x^*) + G_{wL}(x^*)$. By continuity of the marriage market masses, there exists an open interval around x^* such that for any x in the open interval, $G_{mH}(x) < G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$. Define $x_s \equiv \sup\{x : G_{mH}(x) = G_{wH}(x) + G_{wh}(x) + G_{wL}(x)\}$. By continuity of the masses, the set $\{x : G_{mH}(x) = G_{wH}(x) + G_{wh}(x) + G_{wL}(x)\}$ is compact, and supremum exists. For example, in Figure A1, $x_s = \sup\{x_{1.3}\} = x_{1.3}$, and in Figure A2, $x_s = \sup\{x_n, x_{1.3}\} = x_{1.3}$. If $G_{mH}(x) < G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$ for all x, the set is empty, then let $\sup \emptyset \equiv 0$. $x_s < x_{1.5}$.
   (a) For any $x \in [0, x_s]$, $\min \phi_1(x) \geq x_{1.5} > x$, so no fixed point in $[0, x_s]$: Since $G_{mH}(x_s) = G_{wH}(x_s) + G_{wh}(x_s) + G_{wL}(x_s) > G_{wH}(x_s) + G_{wh}(x_s)$, $G_{mH}(x)$ is monotonically decreasing, and $G_{wH}(x) + G_{wh}(x)$ is monotonically increasing for any $x \in [0, x_{1.5}]$, $\min \phi_1(x) > x_{1.5}$.
   (b) For any $x \in (x_s, x^*)$, $\min \phi_1(x) \geq x^*$, so no fixed point in (x_s, x^*): Since $x > x_s$, $G_{mH}(x) < G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$. Since $G_{mH}(x)$ is monotonically decreasing and $G_{wH}(x) + G_{wh}(x)$ and $G_{wH}(x)$ are monotonically increasing for any $x < x^*$, $\phi_1(x)$ is monotonically decreasing in this interval, so $x_s < x < x^*$ but $\min \phi_1(x) > x^*$.
   (c) For any $x \in (x^*, x_{1.5}]$, $\max \phi_1(x) < x^* < x_{1.5}$, so no fixed point in $(x^*, x_{1.5}]$: Since $G_{mH}(x)$ is monotonically decreasing and $G_{wH}(x) + G_{wh}(x)$ and $G_{wH}(x)$ are monotonically increasing for any $x < x^*$, $\phi_1(x)$ is monotonically decreasing in this interval, so $x_{1.5} > x > x^*$ but $\max \phi_1(x) < x^*$.
   (d) For any $x \in (x_{1.5}, 1]$, $\max \phi_1(x) \leq x_{1.5}$, so no fixed point in $(x_{1.5}, 1]$: $G_{mH}(x) < G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$, so $\max \phi_1(x) \leq x_{1.5}$.

2. Suppose there is a fixed point $x^* = x_{1.5}$:

$$G_{wH}(x_{1.5}) \in [G_{wH}(x_{1.5}) + G_{wh}(x_{1.5}), G_{wH}(x_{1.5}) + G_{wh}(x_{1.5}) + G_{wL}(x_{1.5})]$$

   (a) For any $x \in [0, x_{1.5})$, $\min \phi_1(x) \geq x_{1.5}$, so no fixed point in $[0, x_{1.5})$:

$$G_{mH}(x) > G_{mH}(x_{1.5}) \geq G_{wH}(x^*) + G_{wh}(x^*) \geq G_{wH}(x) + G_{wh}(x),$$

   where the inequalities follow from strict monotonicities of $G_{mH}(x)$ and $G_{wH}(x) + G_{wh}(x)$ between 0 and $x_{1.5}$, so $\min \phi_1(x) \geq x_{1.5}$.
   (b) For any $x \in (x_{1.5}, 1]$, $\max \phi_1(x) \leq x_{1.5}$, so no fixed point in $(x_{1.5}, 1]$:

$$G_{mH}(x) < G_{mH}(x_{1.5}) \geq G_{wH}(x^*) + G_{wh}(x^*) \geq G_{wH}(x) + G_{wh}(x),$$

   where the inequalities follow from strict monotonicities of $G_{mH}(x)$ and $G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$ between $x_{1.5}$ and 1, so $\max \phi_1(x) \leq x_{1.5}$.

Table 4: Directions of changes in marriage payoff differences and marriage-market masses as x increases: $s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$.

	$s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$		
	$x \le x_{2.3}$	$x_{2.3} \le x \le x_{2.5}$	$x \ge x_{2.5}$
$(v_{mH} - v_{mL})_x$	↓	↓	↓
$(v_{wH} - v_{wL})_x$	↑	-	-
$(v_{wh} - v_{wl})_x$	-	-	↑
$(v_{wL} - v_{wl})_x$	-	↑	↑
$p_m(x)$	↑	↑	↑
$p_{w1}(x)$	↓	-	-
$p_{w2}(x)$	-	↑	↑*
$G_{mH}(x)$	↓	↓	↓
$G_{wH}(x)$	↑	↓	↓
$G_{wH}(x) + G_{wL}(x)$	-	↑	↑
$G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$	-	↑	↑

3. Suppose there is a fixed point $x^* > x_{1.5}$. $G_{mH}(x^*) \ge G_{wH}(x^*) + G_{wh}(x^*) + G_{wL}(x^*)$.

   (a) For any $x \in [0, x_{1.5})$, $\min \phi_1(x) \ge x_{1.5}$: $G_{mH}(x) > G_{mH}(x_{1.5}) > G_{mH}(x^*) \ge G_{wH}(x^*) + G_{wh}(x^*) + G_{wL}(x^*) > G_{wH}(x_{1.5}) + G_{wh}(x_{1.5}) + G_{wL}(x_{1.5}) > G_{wH}(x_{1.5}) + G_{wh}(x_{1.5}) > G_{wH}(x) + G_{wh}(x)$ where $G_{mH}(x)$ and $G_{wH}(x) + G_{wh}(x)$ are strictly increasing on $[0, x_{1.5}]$ and $G_{mH}(x)$ and $G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$ are strictly decreasing on $[x_{1.5}, x^*]$. $G_{mH}(x) > G_{wH}(x) + G_{wh}(x)$ implies $\min \phi_1(x) \ge x_{1.5}$.

   (b) For any $x \in [x_{1.5}, x^*)$, $\min \phi_1(x) > x^*$: $G_{mH}(x) > G_{mH}(x^*) \ge G_{wH}(x^*) + G_{wh}(x^*) + G_{wL}(x^*) > G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$ implies $\phi_1(x)$ is monotonically decreasing on $[x_{1.5}, x^*]$, so $\min \phi_1(x) \ge x^*$.

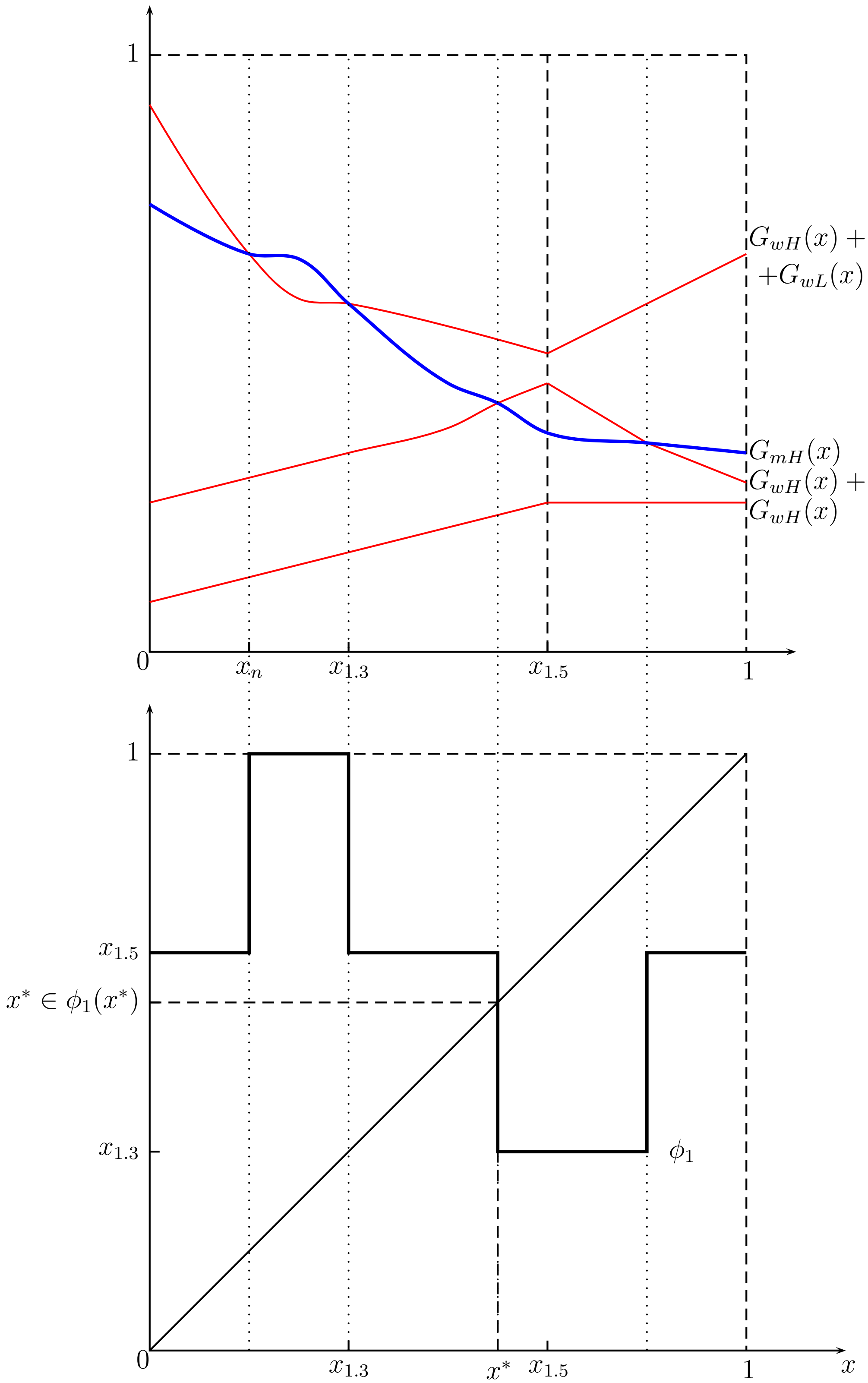
   (c) For any $x \in (x^*, 1]$, $\max \phi_1(x) \le x^*$, so no fixed point: $\phi_1(x)$ is upper bounded by x^* $G_{mH}(x)$ monotonically decreases and $G_{wH}(x)+G_{wh}(x)+G_{wL}(x)$ monotonically increases, so $\max \phi_1(x) \le x^*$.

□

Case 2. Second, suppose $s_{Hh} + s_{LL} \le s_{HL} + s_{Lh}$. Analogous to defining ϕ_1 in Case 1, define

$$\phi_2(x) \equiv \begin{cases} \{0\}, & 0 \le G_{mH}(x) < G_{wH}(x) \\ [0, x_{2.3}], & G_{mH}(x) = G_{wH}(x) \\ \{x_{2.3}\}, & G_{wH}(x) < G_{mH}(x) < G_{wH}(x) + G_{wL}(x) \\ [x_{2.3}, x_{2.5}], & G_{mH}(x) = G_{wH}(x) + G_{wL}(x) \\ \{x_{2.5}\}, & G_{wH}(x) + G_{wL}(x) < G_{mH}(x) < G_{wH}(x) + G_{wL}(x) + G_{wh}(x) \\ [x_{2.5}, 1], & G_{mH}(x) = G_{wH}(x) + G_{wL}(x) + G_{wh}(x) \\ \{1\}, & G_{wH}(x) + G_{wL}(x) + G_{wh}(x) < G_{mH}(x) \le 1 \end{cases}$$

Figure A2: Constructing $\phi_1(\cdot)$ and finding the unique fixed point $x^* \in \phi_1(x^*)$: an example where $\phi_1(\cdot)$ is not monotonically decreasing but still has a unique fixed point.



Existence and uniqueness of a fixed point of $\phi_2(\cdot)$ can be analogously shown as those of $\phi_1(\cdot)$. To provide some different approaches to the proof, I present the arguments in slightly different ways from the first case, but the arguments are the same in essence.

Claim 2A. $\phi_2(\cdot)$ has a fixed point $x^* \in \phi_2(x^*)$.

Proof of Claim 2A. ϕ_2 is upper-hemicontinuous because $G_{mH}(x)$, $G_{wH}(x)$, $G_{wh}(x)$, and $G_{wL}(x)$ are continuous as $p_m(x)$, $p_{w1}(x)$, and $p_{w2}(x)$ are continuous. $\phi_2(x)$ is non-empty and convex for all x. Therefore, by Kakutani's fixed point theorem, $\phi_2 : [0,1] \twoheadrightarrow [0,1]$ has a fixed point x^* such that $x^* \in \phi_2(x^*)$. □

Claim 2B. $\phi_2(\cdot)$ has a unique fixed point $x^* \in \phi_2(x^*)$.

Proof of Claim 2B. Summarized in Table 4 are the directions of changes in the marriage payoff differences, probability cutoffs, and marriage-market masses needed in different steps of the proof. All the directions can be straightforwardly derived from the definitions except for the * sign, which requires some algebra: For $x \in [x_{2.5}, 1]$,

$$\begin{aligned} p_{w2}(x) &= \frac{c_w + (1-r)(v_{wL} - v_{wl})_x}{y_{wH} - y_{wL} + r(v_{wH} - v_{wL})_x + (1-r)(v_{wh} - v_{wl})_x} \\ &= \frac{c_w + (1-r)[(1-\lambda(x))(s_{Lh} - s_{Ll}) + \lambda(x)(s_{Hh} - s_{Hl})]}{k(x) + (1-r)[(1-\lambda(x))(s_{Lh} - s_{Ll} + s_{HL} - s_{Hh}) + \lambda(x)(s_{HL} - s_{Hl})]} \\ &= \frac{c_w + (1-r)[s_{Lh} - s_{Ll} + \lambda(x)(s_{Hh} - s_{Hl} - s_{Lh} + s_{Ll})]}{k(x) + (1-r)[(1-\lambda(x))(s_{Lh} - s_{Ll} + s_{HL} - s_{Hh}) + \lambda(x)(s_{HL} - s_{Hl})]} \end{aligned}$$

where $k(x) \equiv y_{wH} - y_{wL} + r(v_{wH} - v_{wL})_x$ and $\lambda(x) \equiv (x - x_{2.5})/(1 - x_{2.5}) \in [0,1]$. Since $(v_{wH} - v_{wL})_x = s_{HH} - s_{HL}$ is constant, $k(x)$ is constant, and

$$p_{w2}(x) = \frac{\text{constant}_1 + (1-r)\lambda(x)(s_{Hh} - s_{Hl} - s_{Lh} + s_{Ll})}{\text{constant}_2 + (1-r)\lambda(x)(s_{Hh} - s_{Hl} - s_{Lh} + s_{Ll})} < 1$$

strictly increases in x.

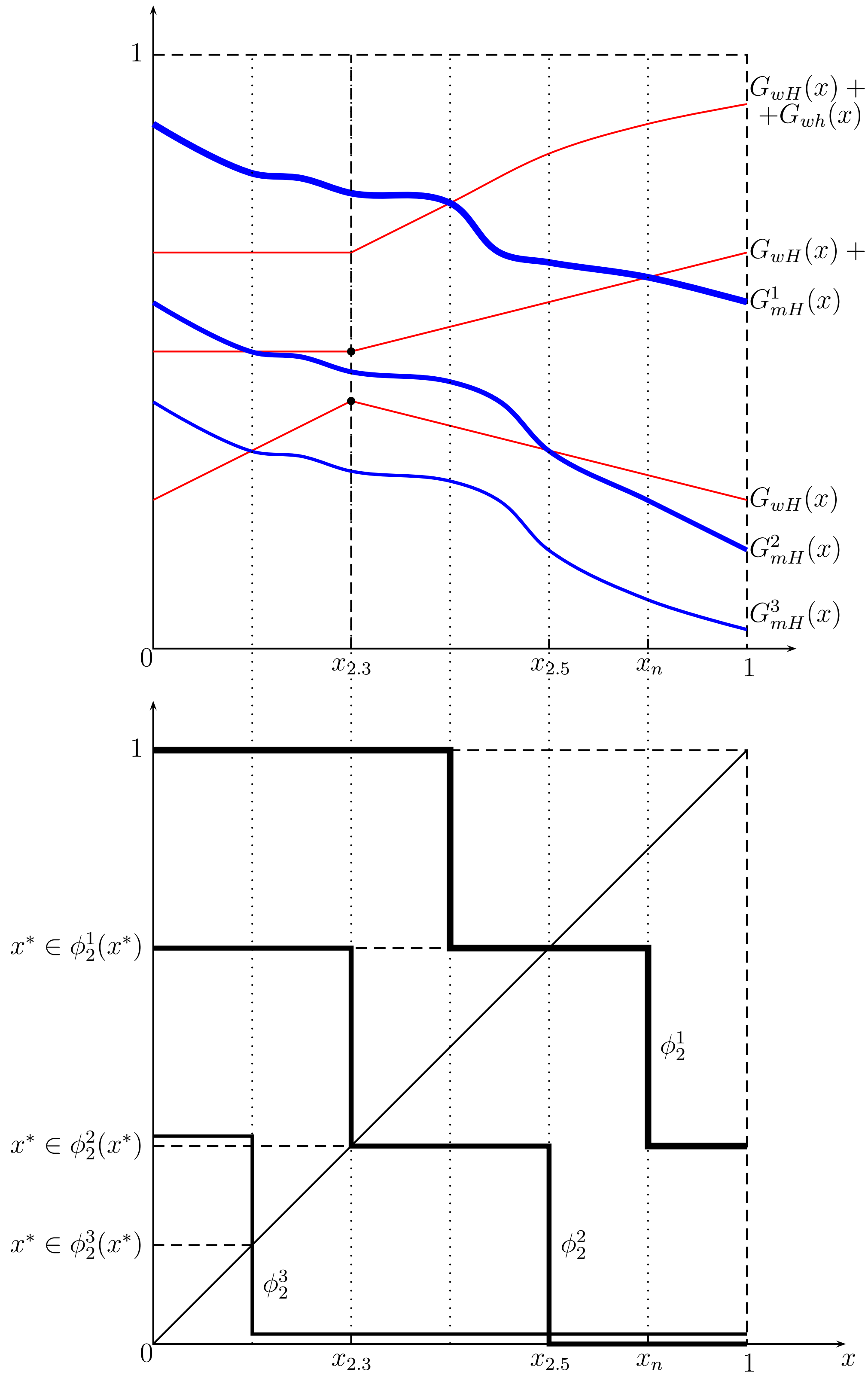
1. $G_{mH}(x_{2.3}) \leq G_{wH}(x_{2.3})$. For any $x \in (x_{2.3}, 1]$,

   $$G_{mH}(x) < G_{mH}(x_{2.3}) \leq G_{wH}(x_{2.3}) < G_{wH}(x_{2.3}) + G_{wL}(x_{2.3}) < G_{wH}(x) + G_{wL}(x),$$

   where the first inequality follows from that $G_{mH}(x)$ is strictly decreasing between $x_{2.3}$ and 1 and the last inequality follows from that $G_{wH}(x) + G_{wL}(x)$ is strictly increasing between $x_{2.3}$ and 1. $G_{mH}(x) < G_{wH}(x) + G_{wL}(x)$ implies $\max \phi_2(x) \leq x_{2.3}$. There can only be a fixed point $x^* \in [0, x_{2.3}]$.

   (a) If $G_{mH}(0) \leq G_{wH}(0)$, then $\phi_2(0) \subseteq [0, x_{2.3}]$, so 0 is a fixed point. For any $x \in (0, x_{2.3}]$, $G_{mH}(x) < G_{mH}(0) \leq G_{wH}(0) < G_{wH}(x)$, where the first inequality follows from that G_{mH} is strictly decreasing in x between 0 and $x_{2.3}$ and the last inequality follows from that G_{wH} is strictly increasing in x between 0 and $x_{2.3}$. $G_{mH}(x) < G_{wH}(x)$ implies $\phi_2(x) = 0$, so there is no fixed point other than 0.

Figure A3: An illustration of constructing $\phi_2(\cdot)$ and finding the unique fixed point $x^* \in \phi_2(x^*)$: $G^1_{mH}(x)$, $G^2_{mH}(x)$, and G^3_{mH} respectively illustrate the three scenarios in Case 2.



(b) If $G_{mH}(0) > G_{wH}(0)$, then coupled with $G_{mH}(x_{2.3}) < G_{wH}(x_{2.3})$, strictly decreasing G_{mH} and strictly increasing G_{wH} on $[0, x_{2.3}]$, the property implies that there is a unique $x^* \in (0, x_{2.3})$ such that $G_{mH}(x^*) = G_{wH}(x^*)$ which implies $\phi_2(x^*) = [0, x_{2.3}]$, and x^* is the unique fixed point of ϕ_2.

2. $G_{wH}(x_{2.3}) < G_{mH}(x_{2.3}) < G_{wH}(x_{2.3}) + G_{wL}(x_{2.3})$.

   (a) By definition of ϕ_2, $\phi_2(x_{2.3}) = \{x_{2.3}\}$. Hence $x_{2.3}$ is a fixed point of ϕ_2.

   (b) For any $x \in [0, x_{2.3})$,

$$G_{mH}(x) > G_{mH}(x_{2.3}) > G_{wH}(x_{2.3}) > G_{wH}(x),$$

   where the first inequality follows from that G_{mH} is strictly decreasing on $[0, x_{2.3})$ and G_{wH} is strictly increasing on $[0, x_{2.3})$. $G_{mH}(x) > G_{wH}(x)$ implies $\min \phi_2(x) \geq x_{2.3}$. Therefore, there is no fixed point in $[0, x_{2.3})$.

   (c) For any $x \in (x_{2.3}, 1]$,

$$G_{mH}(x) < G_{mH}(x_{2.3}) < G_{wH}(x_{2.3}) + G_{wL}(x_{2.3}) < G_{wH}(x) + G_{wL}(x),$$

   where the first inequality follows from that $G_{mH}(x)$ is strictly decreasing between $x_{2.3}$ and 1 and the last inequality follows from that $G_{wH}(x) + G_{wL}(x)$ is strictly decreasing between 0 and $x_{2.3}$. $G_{mH}(x) < G_{wH}(x) + G_{wL}(x)$ implies $\max \phi_2(x) \leq x_{2.3}$. Therefore, there is no fixed point in $(x_{2.3}, 1]$.

3. $G_{mH}(x_{2.3}) \geq G_{wH}(x_{2.3}) + G_{wL}(x_{2.3})$.

   (a) For any $x \in [0, x_{2.3})$,

$$G_{mH}(x) > G_{mH}(x_{2.3}) \geq G_{wH}(x_{2.3} + G_{wL}(x_{2.3}) = G_{wH}(x) + G_{wL}(x),$$

   where the first inequality follows from that G_{mH} is strictly decreasing in x on $[0, x_{2.3})$ and the last equality follows from that $G_{wH}(x) + G_{wL}(x)$ is constant on $[0, x_{2.3}]$. For any x between 0 and $x_{2.3}$, $G_{mH}(x) > G_{wH}(x)$ implies $\min \phi_2(x) \geq x_{2.3}$. Therefore, there is no fixed point in $[0, x_{2.3})$.

   (b) Consider $x \in [x_{2.3}, 1]$. Note that $G_{mH}(x)$ is strictly decreasing, and $G_{wH}(x) + G_{wL}(x)$ and $G_{wH}(x) + G_{wL}(x) + G_{wh}(x)$ are strictly increasing on $[x_{2.3}, 1]$.

      i. If $G_{mH}(x)$ does not intersect either of $G_{wH}(x) + G_{wL}(x)$ and $G_{wH}(x) + G_{wL}(x) + G_{wh}(x)$, then for any $x < 1$,

$$G_{mH}(x) > G_{mH}(1) > G_{wH}(1)+G_{wL}(1)+G_{wh}(1) > G_{wH}(x)+G_{wL}(x)+G_{wh}(x)$$

      where the first inequality follows from that $G_{mH}(x)$ is strictly decreasing on $[x_{2.3}, 1]$ and the last inequality follows from that $G_{wH}(x) + G_{wL}(x) + G_{wh}(x)$ is strictly increasing on $[x_{2.3}, 1]$. The inequality implies that $\phi_1(x) = \{1\}$ for all $x \in [0, 1]$, so 1 is the unique fixed point.

ii. If it does intersect, then there is a fixed point by Claim 2A, and the fixed point $x^* \in [x_{2.3}, 1]$ by the argument immediately above. It remains to show x^* is the only fixed point, or in other words, there is no other fixed point in $[x_{2.3}, 1]$.
First, check any $x \in [x_{2.3}, x^*)$. At least one of the two conditions holds: $G_{mH}(x^*) \geq G_{wH}(x^*) + G_{wL}(x^*) + G_{wh}(x^*)$ and $G_{mH}(x^*) \geq G_{wH}(x^*) + G_{wL}(x^*)$. Since $G_{mH}(x)$ is strictly decreasing and $G_{wH}(x)+G_{wL}(x)+G_{wh}(x)$ and $G_{wH}(x)+G_{wL}(x)$ are strictly decreasing on $[x_{2.3}, 1]$, for any $x \in [x_{2.3}, x^*)$, $G_{mH}(x^*) \geq G_{wH}(x^*) + G_{wL}(x^*) + G_{wh}(x^*)$ implies $G_{mH}(x) > G_{mH}(x^*) \geq G_{wH}(x^*)+G_{wL}(x^*)+G_{wh}(x^*) > G_{wH}(x)+G_{wL}(x)+G_{wh}(x)$, and $G_{mH}(x^*) \geq G_{wH}(x^*) + G_{wL}(x^*)$ implies $G_{mH}(x) > G_{mH}(x^*) \geq G_{wH}(x^*) + G_{wL}(x^*) > G_{wH}(x) + G_{wL}(x)$. For any $x \in [x_{2.3}, x^*)$, $\min \phi_2(x) \geq x^*$. Hence, there is no fixed point in $[x_{2.3}, x^*]$.
Define $x_s \equiv \inf\{x : G_{mH}(x) = G_{wH}(x) + G_{wL}(x)\}$, $\equiv 1$ if the set $\{x : G_{mH}(x) = G_{wH}(x) + G_{wL}(x)\}$ is empty (for example, in Figure A3, $x_s = x_{2.5}$ for $G^1_{mH}(x)$; $x_s = x_n$ for $G^2_{mH}(x)$; $x_s = 1$ for $G^3_{mH}(x)$).
Second, check any $x \in (x^*, x_s)$. At least one of the two statements holds: $G_{mH}(x^*) \leq G_{wH}(x^*) + G_{wL}(x^*)$ and $G_{mH}(x^*) \leq G_{wH}(x^*) + G_{wL}(x^*) + G_{wh}(x^*)$. Since $G_{mH}(x) < G_{mH}(x^*)$ and $G_{wH}(x) + G_{wL}(x)$ and $G_{wH}(x) + G_{wL}(x)+G_{wh}(x)$ are strictly decreasing on $(x^*, x_s) \subseteq (x_{2.3}, 1)$, the statement that holds at x^* still holds for any $x \in (x^*, x_s)$. As a result, $\max \phi_2(x) \leq x^*$ for $x \in (x^*, x_s)$.
Third and finally, check any $x \in [x_s, 1]$. Since $G_{mH}(x_s) = G_{wH}(x_s)$,

$$G_{mH}(x) < G_{mH}(x_s) = G_{wH}(x_s) < G_{wH}(x_s) + G_{wL}(x_s) < G_{wH}(x) + G_{wL}(x),$$

where the first and last inequalities follow from strict monotonicity of $G_{mH}(x)$ and $G_{wH}(x) + G_{wL}(x)$ on $[x_{2.3}, 1]$. $G_{mH}(x) < G_{wH}(x) + G_{wL}(x)$ implies $\max \phi_2(x) \leq x_{2.3}$.

□

In summary, an upper-hemicontinuous and convex-valued map from an index representing a set of stable marriage payoff differences to a set of indices representing stable marriage payoff differences is constructed to show a unique fixed point. □

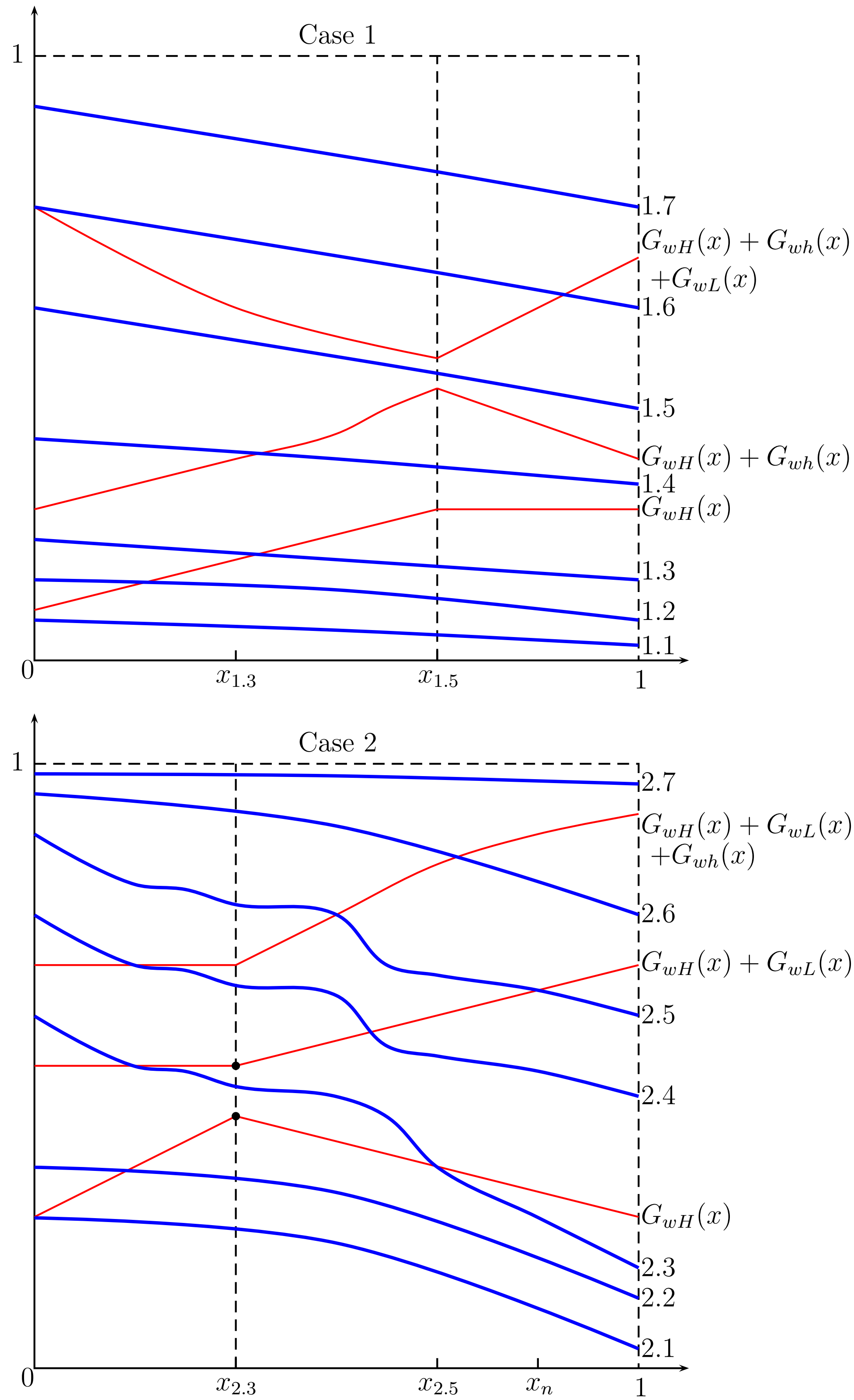
Characterization of the Equilibrium

Four marriage payoff differences, $v^*_{mH} - v^*_{mL}$, $v^*_{wH} - v^*_{wL}$, $v^*_{wh} - v^*_{wl}$, and $v^*_{wL} - v^*_{wl}$, together characterize an equilibrium. The index x between 0 and 1 defined in the proof of equilibrium existence and uniqueness uniquely characterizes the four equilibrium marriage payoff differences given $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$ or $s_{Hh} + s_{LL} \leq s_{HL} + s_{Lh}$. To characterize the equilibrium, it suffices to solve for the equilibrium index representing the equilibrium marriage payoff differences. Namely, in equilibrium,

$$v^*_{mH} - v^*_{mL} = (1 - x^*)(s_{HH} - s_{LH}) + x^*(s_{Hl} - s_{Ll})$$

where x^* is determined as follows, and the other three equilibrium marriage payoff differences are the accompanying marriage payoff differences in Table 1.

Figure A4: Characterization of the equilibrium: all 14 possibilities.



1. $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$.

   1.1 $G_{mH}(0) < G_{wH}(0)$: $x^* = 0$.

   1.2 $G_{mH}(0) \geq G_{wH}(0)$ and $G_{mH}(x_{1.3}) < G_{wH}(x_{1.3})$: $x^* \in [0, x_{1.3}]$ is the unique solution to $G_{mH}(x) = G_{wH}(x)$.

   1.3 $G_{wH}(x_{1.3}) \leq G_{mH}(x_{1.3}) < G_{wH}(x_{1.3}) + G_{wh}(x_{1.3})$: $x^* = x_{1.3}$.

   1.4 $G_{mH}(x_{1.3}) \geq (G_{wH} + G_{wh})(x_{1.3})$ and $G_{mH}(x_{1.5}) < (G_{wH} + G_{wh})(x_{1.5})$: $x^* \in [x_{1.3}, x_{1.5}]$ is the unique solution to $G_{mH}(x) = G_{wH}(x) + G_{wh}(x)$.

   1.5 $G_{wH}(x_{1.5}) + G_{wh}(x_{1.5}) \leq G_{mH}(x_{1.5}) < G_{wH}(x_{1.5}) + G_{wh}(x_{1.5}) + G_{wL}(x_{1.5})$: $x^* = x_{1.5}$.

   1.6 $G_{mH}(x_{1.5}) \geq G_{wH}(x_{1.5}) + G_{wh}(x_{1.5}) + G_{wL}(x_{1.5})$ and $G_{mH}(1) < G_{wH}(1) + G_{wh}(1) + G_{wL}(1)$: $x^* \in [x_{1.5}, 1]$ is the unique solution to $G_{mH}(x) = G_{wH}(x) + G_{wh}(x) + G_{wL}(x)$.

   1.7 $G_{mH}(1) \geq G_{wH}(1) + G_{wh}(1) + G_{wL}(1)$: $x^* = 1$.

2. $s_{Hh} + s_{LL} \leq s_{HL} + s_{Lh}$.

   2.1 $G_{mH}(0) < G_{wH}(0)$: $x^* = 0$.

   2.2 $G_{mH}(0) \geq G_{wH}(0)$ and $G_{mH}(x_{2.3}) < G_{wH}(x_{2.3})$: $x^* \in [0, x_{2.3}]$ is the unique solution to $G_{mH}(x) = G_{wH}(x)$.

   2.3 $G_{wH}(x_{2.3}) \leq G_{mH}(x_{2.3}) < G_{wH}(x_{2.3}) + G_{wL}(x_{2.3})$: $x^* = x_{2.3}$.

   2.4 $G_{mH}(x_{2.3}) \geq (G_{wH} + G_{wL})(x_{2.3})$ and $G_{mH}(x_{2.5}) < (G_{wH} + G_{wL})(x_{2.5})$: $x^* \in [x_{2.3}, x_{2.5}]$ is the unique solution to $G_{mH}(x) = G_{wH}(x) + G_{wL}(x)$.

   2.5 $G_{wH}(x_{2.5}) + G_{wL}(x_{2.5}) \leq G_{mH}(x_{2.5}) < G_{wH}(x_{2.5}) + G_{wL}(x_{2.5}) + G_{wh}(x_{2.5})$: $x^* = x_{2.5}$.

   2.6 $G_{mH}(x_{2.5}) \geq G_{wH}(x_{2.5}) + G_{wL}(x_{2.5}) + G_{wh}(x_{2.5})$ and $G_{mH}(1) < G_{wH}(1) + G_{wL}(1) + G_{wh}(1)$: $x^* \in [x_{2.5}, 1]$ is the unique solution to $G_{mH}(x) = G_{wH}(x) + G_{wL}(x) + G_{wh}(x)$.

   2.7 $G_{mH}(1) \geq G_{wH}(1) + G_{wL}(1) + G_{wh}(1)$: $x^* = 1$.

Figures

Figure A5 illustrates men's average logged income by age at marriage. Figure A6 illustrates women's average logged income by age at marriage. Figure A7 illustrates husband's average logged income by wife's age at marriage, fixing men's birth cohort. Figure A8 illustrates husband's average logged income by wife's age at marriage, fixing women's birth cohort.

Data Sources

The figures were plotted for 6 different birth cohorts (1910-19, 1920-29, 1930-39, 1940-49, 1950-59, and 1960-69). The data for these cohorts come from 4 different data sets (the 1960, 1970, 1980 U.S. Censuses and the 2010 ACS). See the IPUMS website for detailed descriptions of the datasets named specifically below, https://usa.ipums.org/usa/sampdesc.shtml.

The 1900-09 birth cohort and 1910-19 birth cohort's data are from the 1960 U.S. Census 1% sample. The 1% sample was chosen over the 5% sample because the latter is currently missing the strata variable for survey weighting as the data set is being reconstructed by IPUMS. The age at marriage is identified with the variable `agemarr`, where the respondents were asked how old they were when they first got married.

The 1920-29 birth cohort's data are from the 1970 U.S. Census 1% Form 1 Metro Sample. This dataset from that census is used because other data sets from the 1970 Census do not contain pertinent information on age at marriage. The word "Metro" in the dataset's name does not imply it is only a sample of persons living in metropolitan areas. Rather it indicates that the metropolitan geographic identifiers are available. The age at marriage is identified with the variable `agemarr`, where the respondents were asked how old they were when they first got married.

The 1930-39 birth cohort's data are from the 1980 U.S. Census 5% State sample. This dataset was chosen over the others for having a larger sample size than the 1% samples while still having all of the variables of interest. The age at marriage is identified with the variable `agemarr`, where the respondents were asked how old they were when they first got married.

The 1990 U.S. Census and the 2000 American Community Survey (A.C.S., the replacement to the U.S. Census) did not contain detailed questions pertaining to marriage. Beginning in 2008, and including the 2010 A.C.S., the year of current marriage was asked, from which straightforward arithmetic can determine the age at first marriage. Because the variable `yrmarr` corresponds to the year of the current marriage, only persons married once are assigned an age at first marriage, to be consistent with `agemarr`. If a husband was married twice, but the wife has been only married once, then the husband's income is still used for the graphs concerning women's spousal income. The 2010 A.C.S. is then used for 1940-49, 1950-59, and 1960-69 birth cohorts (ages 61-70, ages 51-60, and ages 41-50, respectively).

The 1970-79 birth cohort's data are ages 35-44 from the 2014 A.C.S., which contained the same variables and therefore same calculations as cohorts examined in the 2010 A.C.S..

For all datasets, spousal data was extracted from IPUMS. The Censuses (and A.C.S.) ask how each member of a household is related to the head of household. From there, IPUMS was able to infer which pairs of people were married to each other. Using the "attach characteristics" feature from IPUMS, pertinent spousal data was extracted. Spouses in these graphs then must have been reported to reside within the same household to appear in the calculations for these graphs.

Data Calculations

Survey weights were used for all estimations of means and their confidence intervals, using the Stata command

```
svyset cluster [pweight=perwt], strata(strata)
```

where `perwt` and `strata` are variables extracted via IPUMS.

Incomes were converted into 2015 USD using table 24 in the CPI Detailed report from May 2016 (http://www.bls.gov/cpi/cpid1605.pdf). More specifically, the annual CPI-U figures were used, keeping in mind that the incomes reported in the Census and A.C.S. are from the previous calendar year.

For women's own and men's own income, `inctot`, the means and 95% confidence intervals are computed for each age at first marriage bin. For the 1910-19, 1920-29, 1930-39, and 1960-69 cohorts, only incomes earned by individuals aged between 41-50 as reported in the Census or A.C.S. are included in the means. For the 1940-49 and 1950-59 cohorts which both use the 2010 A.C.S, the person must be aged 61-70 or 51-60, respectively, to be included in the calculation of means. This is solely due to a lack of data available to measure those birth cohorts' incomes between ages 41-50 by age at first marriage.

For means of women's spousal income, the woman must satisfy the same age requirements as above, but their spouses can be of any age.

The specific command for calculating the bin-averages is

```
svy, sub(‘yAgeReq’ if ‘xBinVar’==‘j’ & sex==‘sexValue’) :  mean ‘yVar’
```

where `svy` is the prefix for "survey" indicating that survey weights are being used, and the subpopulation would be persons meeting the age requirement if they also have the proper age at first marriage and are the proper sex, and the mean of `‘yVar’` is being calculated. The estimated mean, along with the confidence interval, is then saved for each age at first marriage bin and then plotted.

Figure A5: Men's Log Income by Age at Marriage.

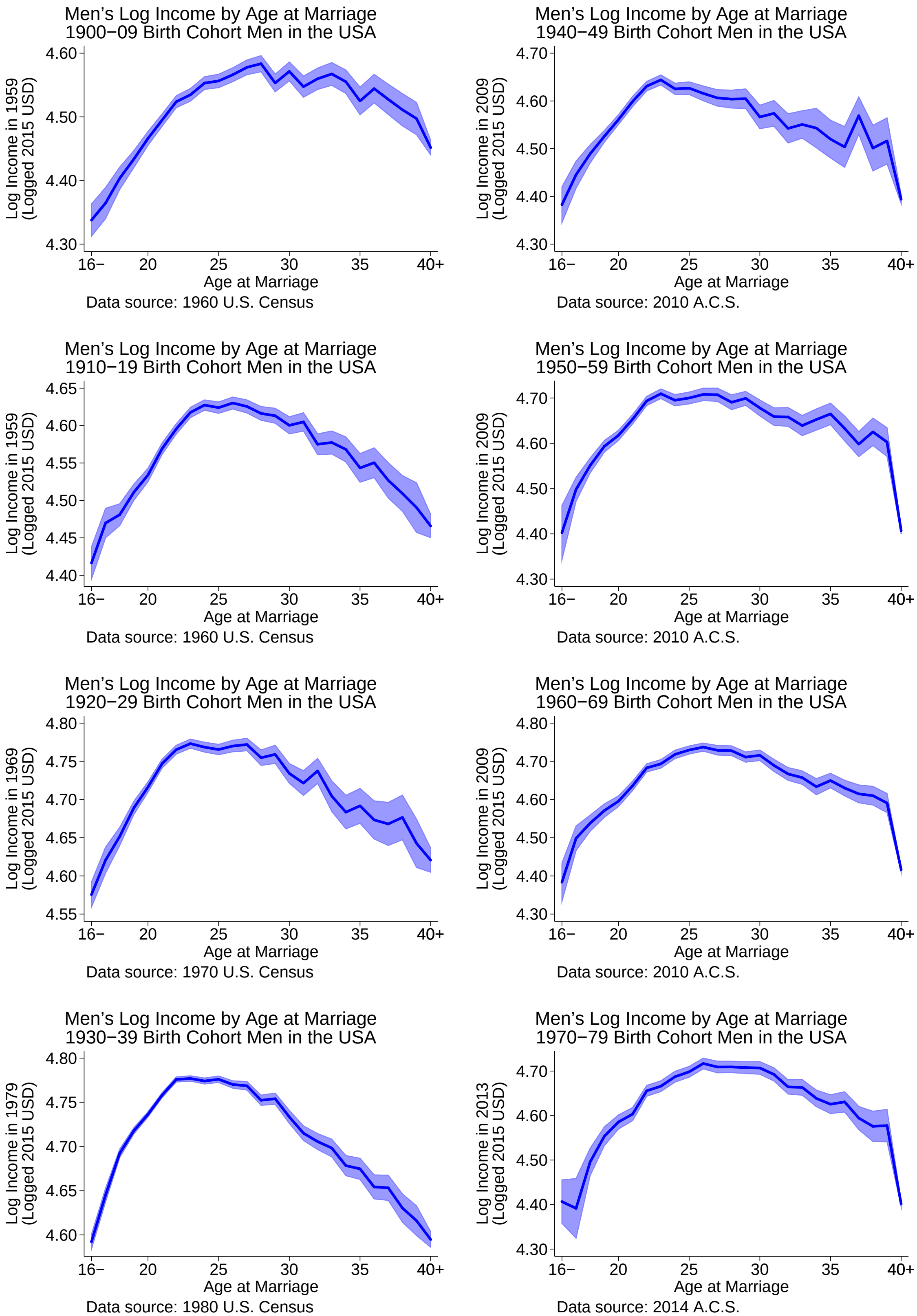


Figure A6: Women's Log Income by Age at Marriage.

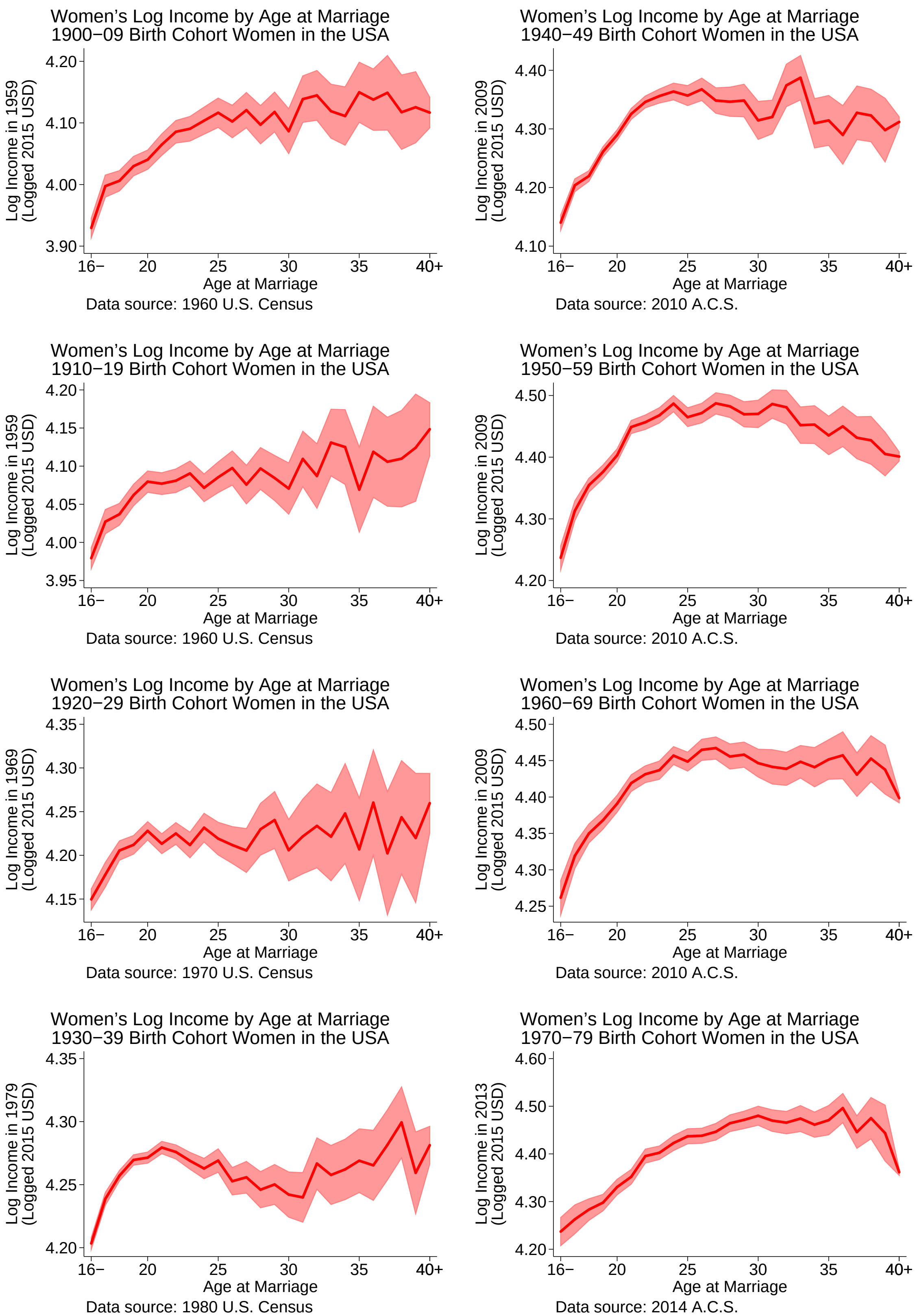


Figure A7: Husband's Log Income by Wife's Age at Marriage: Fixing Women's Age.

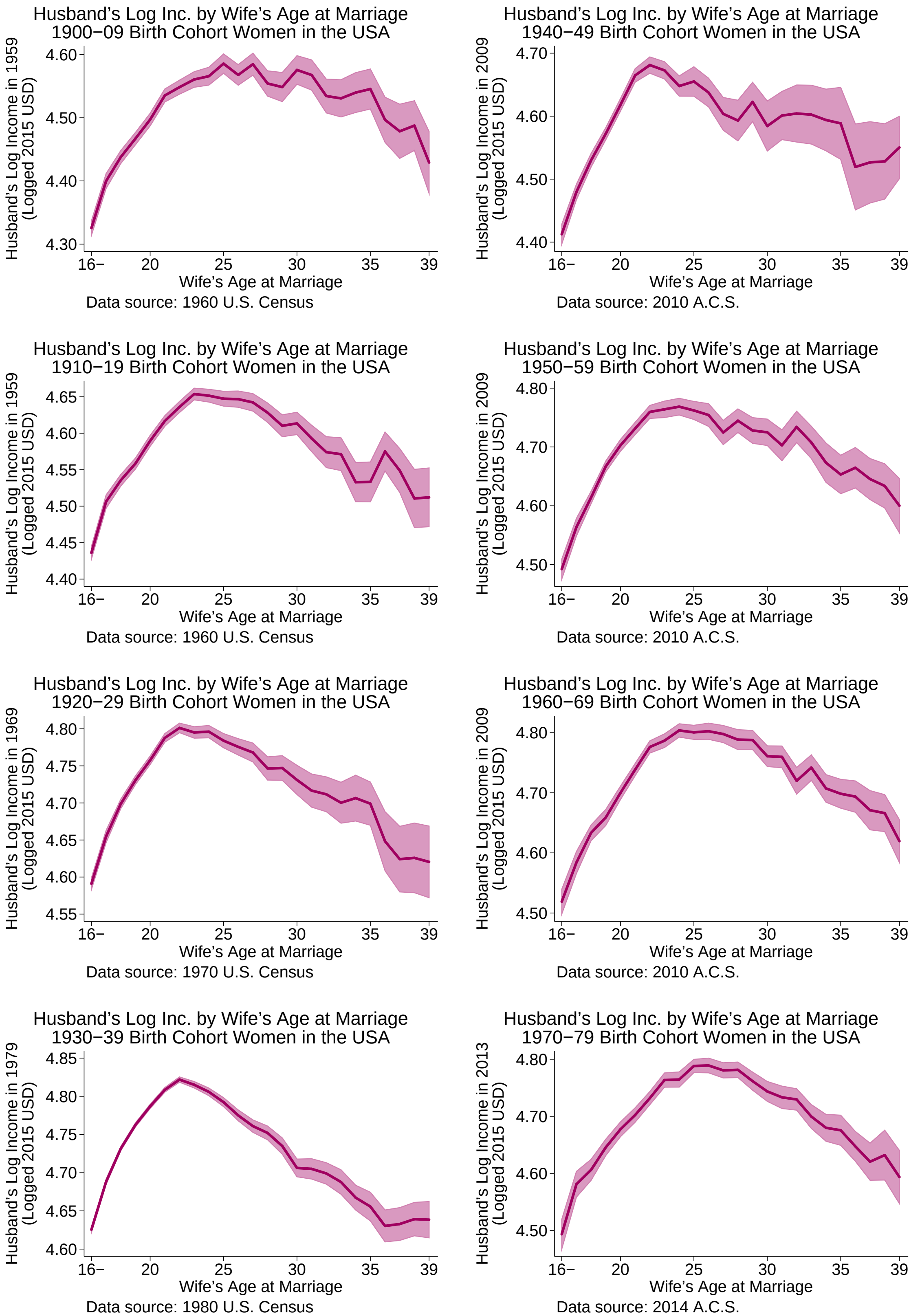


Figure A8: Husband's Log Income by Wife's Age at Marriage: Fixing Men's Age.