Pre-Matching Gambles

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Pre-Matching Gambles

- Risky investments before matching market.
  - Agents make investments with stochastic returns to change their matching characteristics.
  - They match based on realized characteristics in a matching market.
- Examples
  - College majors $\to$ labor market
  - Careers paths $\to$ marriage market
  - Financial portfolios $\to$ investors market

Main Results

1. Pre-matching gambles are prevalent and can be extreme.
2. The competitive organization of the matching market (stability reinterpreted) encourages gambles.
   - The gamble-inducing effect is independent of the shape of the surplus function (e.g. supermodularity) and the distributions of matching characteristics.
3. Implications
   - Risk-taking is not necessarily welfare-reducing: an example with one inefficient no-gambling equilibrium and another efficient gambling equilibrium.
   - Explains gender differences in risky occupational choices and marriage timing.

Contributions to Literature

1. The first paper to study equilibrium pre-matching investments with stochastic returns.
   - Cole et al. (2001, JET), Dizdar (2013), Nöldeke and Samuelson (2015, Ecta); Chade and Lindenlaub (2015).
2. Provides a new reason for gambling: matching market.
   - Smith (1776), Friedman and Savage (1948, JPE), Friedman (1953, JPE), Rubin and Paul (1979, EI), Robson (1992, Ecta), Robson (1996, GEB), Rosen (1997, JoLE), Becker et al. (2005, JPE).
3. Applications to efficiency, inequality, progressive tax, and occupations and marriage.

A Motivating Example

- Mass 1 of men, $x_m \sim \text{Unif}[0, 1]$
- Mass 1 of women, $x_w \sim \text{Unif}[0, 1]$
- Surplus $s(x_m, x_w) = x_m x_w$
- Stable outcome (stable matching and payoffs)

$$v_m(x_m) + v_w(x_w) = x_m x_w \quad \text{if } x_m \text{ and } x_w \text{ are matched}$$

$$v_m(x_m) + v_w(x_w) \geq x_m x_w \quad \text{for any } x_m \text{ and } x_w$$

Four-Player Stable Outcome

- A characteristic 1 man and a characteristic 2 man
- A characteristic 1 woman and a characteristic 2 woman
- Surplus is $x_m x_w$
- Stable outcome
  - Matching: 2 matches with 2, 1 matches with 1
  - Payoffs: 2s get 2 and 1s get 0.5
  - An unmatched pair (e.g. a 2 man and a 1 woman): $2 + 0.5 > 2 \times 1$.

Gambles Preferred

- **Stable Matching**: Each x_m man is matched with $x_w = x_m$ woman.
  - Supermodular surplus $x_m x_w$ leads to positive-assortative matching.
- **Stable Payoffs**: x_m and $x_w = x_w$ produce and divide surplus x_m^2, $v_m(x) = v_w(x) = \frac{x^2}{2}$.
- The payoff functions are convex.
  1. .5 prefers gamble $\frac{1}{2} \circ .4 + \frac{1}{2} \circ .6$ ($u = \frac{1}{2}\frac{.4^2}{2} + \frac{1}{2}\frac{.6^2}{2} = .13$) to no gamble ($u = \frac{.5^2}{2} = .125$).
  2. .5 doubles utility by switching to an extreme gamble $\frac{1}{2} \circ 0 + \frac{1}{2} \circ 1$ ($u = \frac{1}{2}\frac{1^2}{2} + \frac{1}{2}\frac{0^2}{2} = .25$) from no gamble.
  3. Moderately risk-averse agents prefer to take unfair gambles.

Generalized Motivating Example

- Measure μ_m describes $x_m \in \mathbb{R}$
- Measure μ_w describes $x_w \in \mathbb{R}$
- Surplus $s(x_m, x_w) = A + Bx_m x_w$, $A > 0$, $B \in \mathbb{R}$
- Stable outcome (stable matching and payoffs)

$$v_m(x_m) + v_w(x_w) = s(x_m, x_w) \quad \text{if } x_m \text{ and } x_w \text{ are matched}$$

$$v_m(x_m) + v_w(x_w) \geq s(x_m, x_w) \quad \text{for any } x_m \text{ and } x_w$$

Gamble Preferred

- $\widehat{x}_m$ vs fair extreme gamble $p \circ \overline{x}_m + (1-p) \circ \underline{x}_m$.
- Suppose $\widehat{x}_m$ matches with $\widehat{x}_w$,

$$v_m(\widehat{x}_m) = A + B\widehat{x}_m\widehat{x}_w - v_w(\widehat{x}_w).$$

- By stability, if $\overline{x}_m$ and $\underline{x}_m$ bargain with $\widehat{x}_w$,

$$v_m(\overline{x}_m) \geq A + B\overline{x}_m\widehat{x}_w - v_w(\widehat{x}_w).$$

$$v_m(\underline{x}_m) \geq A + B\underline{x}_m\widehat{x}_w - v_w(\widehat{x}_w).$$

- Since $\widehat{x}_m = p\overline{x}_m + (1-p)\underline{x}_m$,

$$\begin{aligned} & pv_m(\overline{x}_m) + (1-p)v_m(\underline{x}_m) - v_m(\widehat{x}_m) \\ \geq \quad & p[A + B\overline{x}_m\widehat{x}_w - v_w(\widehat{x}_w)] \\ & +(1-p)[A + B\underline{x}_m\widehat{x}_w - v_w(\widehat{x}_w)] \\ & -[A + B\widehat{x}_m\widehat{x}_w - v_w(\widehat{x}_w)] = 0 \end{aligned}$$

Gambling Phase

- μ_m: men's innate characteristics $\widehat{x}_m \in \widehat{X}_m \subset \mathbb{R}^N$.
- μ_w: women's innate characteristics $\widehat{x}_w \in \widehat{X}_w \subset \mathbb{R}^N$.
- $\widehat{x} \in \widehat{X}_m, \widehat{X}_w$ chooses a gamble γ from the given set $\Gamma(\widehat{x})$,
  - γ_0: degenerate gamble is always available.
  - $\gamma(\cdot|\widehat{x})$ represents probability measure of a gamble.
  - Fair gambles: $\int x d\gamma(x|\widehat{x}) = \widehat{x}$.
- σ_m and σ_w represent gambling choices.

Matching Phase

- Gambles σ_m and σ_w induce μ_m and μ_w.
- Surplus function $s(x_m, x_w)$; singles produce zero.
- Matching market outcome
  - **Matching measure** $\mu(\widetilde{X}_m \times \widetilde{X}_w)$ describes the measure of matches between $x_m \in \widetilde{X}_m$ and $x_w \in \widetilde{X}_w$.
  - **Payoff functions** $v_m : X_m \to \mathbb{R}_+$ and $v_w : X_w \to \mathbb{R}_+$.
- Stable outcome
  1. μ has marginals μ_m and μ_w.
  2. $v_m(x_m) + v_w(x_w) = s(x_m, x_w)$ if $(x_m, x_w) \in \text{supp}(\mu)$.
  3. $v_m(x_m) + v_w(x_w) \geq s(x_m, x_w)$ for any x_m and x_w.
  4. A technical condition: v_m, v_w is defined on X_m, X_w.

Equilibrium

- Primitives of the model: $(\widehat{\mu}_m, \widehat{\mu}_w, \Gamma(\cdot), s)$.
- $(\sigma_m^*, \sigma_w^*, \mu_m^*, \mu_w^*, \mu^*, v_m^*, v_w^*)$ is an equilibrium if
  - Equilibrium strategies σ_m^* and σ_w^* maximize the agents' expected payoffs
  - Equilibrium measures of characteristics μ_m^* and μ_w^* are induced by equilibrium strategies σ_m^* and σ_w^*, and
  - Equilibrium outcome (μ^*, v_m^*, v_w^*) is a stable outcome of equilibrium matching market (μ_m^*, μ_w^*).
- Equilibrium exists (Zhang, 2015).

Reinterpret Stability to Competition

$$v_m(x_m) = s(x_m, x_w) - v_w(x_w) \quad \text{if } (x_m, x_w) \in \text{supp}(\mu)$$

$$v_m(x_m) \geq s(x_m, x_w) - v_w(x_w) \quad \forall x_w \in \text{supp}(\mu_w)$$

$$x_w(x_m) \in \text{argmax}_{x_w \in \text{supp}(\mu_w)}[s(x_m, x_w) - v_w(x_w)].$$

$$\begin{aligned} v_m(x_m) &= s(x_m, x_w(x_m)) - v_w(x_w(x_m)) \\ &= \sup_{x_w \in \text{supp}(\mu_w)} [s(x_m, x_w) - v_w(x_w)]. \end{aligned}$$

Decomposition of Gambling Effects

$$\mathbb{E}[v_m(x_m)] - v_m(\widehat{x}_m)$$

$$\text{let } \widehat{x}_w \equiv x_w(\widehat{x}_m)$$

$$= \mathbb{E}[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(\widehat{x}_m, \widehat{x}_w) - v_w(\widehat{x}_w)]$$

$$= \mathbb{E}[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - \mathbb{E}[s(x_m, \widehat{x}_w) - v_w(\widehat{x}_w)] + \mathbb{E}[s(x_m, \widehat{x}_w) - v_w(\widehat{x}_w)] - [s(\widehat{x}_m, \widehat{x}_w) - v_w(\widehat{x}_w)]$$

$$= \underbrace{\mathbb{E}\left[s(x_m, \widehat{x}_w) \cancel{- v_w(\widehat{x}_w)}\right] - \left[s(\widehat{x}_m, \widehat{x}_w) \cancel{- v_w(\widehat{x}_w)}\right]}_{\text{surplus contribution effect}} + \underbrace{\mathbb{E}\left\{[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(x_m, \widehat{x}_w) - v_w(\widehat{x}_w)]\right\}}_{\text{competitive rematching effect} \geq 0}$$

An Example with Two Equilibria

- Mass 1 of characteristics 2 men.
- Mass 1 of characteristics 2 women.
- Gambling options:2 vs $\frac{1}{2} \circ 1 + \frac{1}{2} \circ 3$.
- Surplus $s(x_m, x_w) = x_m x_w$.

Two Equilibria

1. No-Gambling Equilibrium: No one gambles
   - Mass 1 of $(2, 2)$ matches.
   - $v_m^*(1) = 0$, $v_m^*(2) = 2$, $v_m^*(3) = 4$.
   - $v_w^*(1) = 0$, $v_w^*(2) = 2$, $v_w^*(3) = 4$.
   - $SW^* = (1)(2)(2) = 4$.
2. Gambling Equilibrium: Everyone gambles
   - Mass 0.5 of $(1, 1)$ matches and mass 0.5 of $(3, 3)$ matches.
   - $v_m^*(1) = 0.5$, $v_m^*(2) = 1.5$, $v_m^*(3) = 4.5$.
   - $v_w^*(1) = 0.5$, $v_w^*(2) = 1.5$, $v_w^*(3) = 4.5$.
   - $SW^* = (0.5)(3)(3) + (0.5)(1)(1) = 5$.

Remedy: Progressive Tax

- Problems
  - No-gambling equilibrium is inefficient.
  - Gambling equilibrium creates inequality.
- Progressive Tax can
  - $[0, 1)$: subsidize to 1; $[1, 3)$: no tax; $[3, \infty)$: taxed at 2/3.
- Eliminates the inefficient equilibrium
  - $v^\tau(1) = 1$, $v^\tau(2) = 2$, $v^\tau(3) = 3\frac{1}{3}$.
- Reduces inequality
  - $v^\tau(1) = 1$, $v^\tau(2) = 2$, $v^\tau(3) = 3.5$.
- Generates positive tax revenue
  - $\tau = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot (-0.5) = \frac{1}{2}$

Pre-Marital Occupational Choices

- Newborns every period.
- Each agent $\widehat{x}_m \sim \mu_m$, $\widehat{x}_w \sim \mu_w$ lives for two periods.
- Occupational choice
  - **S**afe $\widehat{x}$.
  - **R**isky $\frac{1}{2} \circ (\widehat{x} + \epsilon) + \frac{1}{2} \circ (\widehat{x} - \epsilon)$ realized at age 2; cost $c_R > 0$.
- Marital timing
  - **E**arly at age 1.
  - **D**elay to age 2; cost $c_D > 0$.
- Surplus $s(x_m, x_w) = x_m x_w$.
- Stationary equilibrium.

Occupational Choice and Marital Timing

- **(S, E)**: $v(\widehat{x})$
- (S, D): $v(\widehat{x}) - c_D$
- (R, E): $v(\frac{1}{2} \circ (\widehat{x} + \epsilon) + \frac{1}{2} \circ (\widehat{x} - \epsilon)) - c_R = v(\widehat{x}) - c_R$
- **(R, D)**: $\frac{1}{2} v(\widehat{x} + \epsilon) + \frac{1}{2} v(\widehat{x} - \epsilon) - c_R - c_D$
- (R, D) $>$ (S, E) if

$$\frac{1}{2} v(\widehat{x} + \epsilon) + \frac{1}{2} v(\widehat{x} - \epsilon) - v(\widehat{x}) > c_R + c_D.$$

Implications

Suppose $c_R + c_D$ is bigger for women than for men.

1. Women tend to choose safe jobs and marry earlier.
2. Men tend to choose risky jobs and marry later.
3. Men end up with bigger income variation.
4. Married men take less risk than unmarried men.
5. Men and women of higher abilities tend to enter risky occupations and marry later.
6. Under gender imbalance, unmarried men may decide to take large gambles.

Conclusion

A new idea

Gary S. Becker <gbecker@uchicago.edu> Sat, Feb 22, 2014 at 2:44 PM
To: Hanzhe Zhang <hanzhe@uchicago.edu>

Hanzhe,
Looked over your paper on gambling. Nicely done.
In my discussion in 301 of gambling, I often use a marriage example. Suppose a good and bad marriage, and by gambling you get the resources to go into a good marriage.The assumption I make is that the net utility from a bad marriage (net of all transfers to spouse, etc) is better when I have low incomes, but worse when I have high incomes. Then I would take a fair gamble; if I lose I get the bad marriage and if I win i get the good marriage. The shift from bad to good marriage makes the net utility function convex.

I believe there is a similarity of this discussion to what you do, but I like that you put it into an equilibrium two-sided matching framework. That is a significant advance over the literature.

I will read more carefully.
Gary Becker

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Next Steps

1. To prove general equilibrium properties.
   - Equilibrium gambles are (constrained) efficient.
   - Efficient gambles form an equilibrium.
   - Constrained efficient gambles form an equilibrium.
   - Techniques used for the deterministic pre-matching investments setting are not applicable.

2. To find more applications/implications.
   - Investors build financial portfolios not only for monetary returns but also to attract future investors.
   - Examples in international trade?
   - Other examples?

THANK YOU!

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