Pre-Matching Gambles

Hanzhe Zhang

Michigan State University

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http://www.hanzhezhang.net/

Pre-Matching Gambles

- Risky investments (gambles) before a matching market.
  - Gambling phase: Players make investments with stochastic returns to change their matching characteristics.
  - Matching phase: They match and divide their surplus based on the realized matching characteristics in a matching market.
- Examples of pre-matching gambles
  - College majors $\rightarrow$ workers-firms labor market
  - Careers $\rightarrow$ men-women marriage market
  - Financial portfolios $\rightarrow$ entrepreneurs-investors market

Main Results

1. The competitive organization of the matching market encourages gambles.
   - The gamble-inducing effect is independent of the shape of the surplus function (e.g. surplus supermodularity), degree of utility transferability, and the distributions of matching characteristics.
2. There could be multiple equilibria.
   - An efficient equilibrium with income inequality.
   - An inefficient equilibrium with income equality.
   - A carefully designed tax scheme yields a unique efficient equilibrium with reduced income inequality.
3. Explains gender differences in occupational choices and marriage timing.

Contributions

1. The first to study equilibrium pre-matching investments with stochastic returns.
    - Cole et al. (2001, JET), Dizdar (2013), Nöldeke and Samuelson (2015, Ecta); Chade and Lindenlaub (2015).
2. Provides a new reason for gambling: matching market.
    - Smith (1776), Friedman and Savage (1948, JPE), Friedman (1953, JPE), Rubin and Paul (1979, EI), Robson (1992, Ecta), Robson (1996, GEB), Rosen (1997, JoLE), Becker et al. (2005, JPE).
3. Applications to efficiency, inequality, and tax, and to occupational choices and the marriage market.

A Motivating Example

- Mass 1 of men, $x_m \sim \text{Unif}[0,1]$
- Mass 1 of women, $x_w \sim \text{Unif}[0,1]$
- Surplus $s(x_m, x_w) = x_m x_w$
- Stable outcome (stable matching and payoffs)

$$v_m(x_m) + v_w(x_w) = x_m x_w \quad \text{if } x_m \text{ and } x_w \text{ are matched}$$

$$v_m(x_m) + v_w(x_w) \geq x_m x_w \quad \text{for any } x_m \text{ and } x_w$$

Four-Player Stable Outcome

- Four players: A type 1 man and a type 2 man, a type 1 woman and a type 2 woman.
- Surplus is $x_m x_w$ (e.g. a type 1 man and a type 1 woman generate surplus $1 \times 1 = 1$).
- Stable outcome
  - Matching: 2 matches with 2, 1 matches with 1
  - Payoffs: 2s get 2 and 1s get 0.5
  - The type 2 man and the type 1 woman who are not married to each other do not want to marry each other,

$$v_m(2) + v_w(1) = 2 + 0.5 > 2 \times 1 = s(2, 1).$$

Gambles Preferred

- **Stable Matching**: Each x_m man is matched with $x_w = x_m$ woman.
- **Stable Payoffs**: x_m and $x_w = x_m$ produce and divide surplus x_m^2, $v_m(x) = v_w(x) = \frac{x^2}{2}$.
- The payoff functions are convex.
  1. .5 prefers gamble $\frac{1}{2} \circ .4 + \frac{1}{2} \circ .6$ $(u = \frac{1}{2}\frac{.4^2}{2} + \frac{1}{2}\frac{.6^2}{2} = .13)$ to no gamble $(u = \frac{.5^2}{2} = .125)$.
  2. .5 doubles utility by switching to an extreme gamble $\frac{1}{2} \circ 0 + \frac{1}{2} \circ 1$ $(u = \frac{1}{2}\frac{1^2}{2} + \frac{1}{2}\frac{0^2}{2} = .25)$ from no gamble.
  3. Moderately risk-averse agents prefer to take unfair gambles.

Gambling Phase

- Measure $\widehat{\mu}_m$ of men's innate $\widehat{x}_m \in \widehat{X}_m \subset \mathbb{R}^{N_m}$.
- Measure $\widehat{\mu}_w$ of women's innate $\widehat{x}_w \in \widehat{X}_w \subset \mathbb{R}^{N_w}$.
- $\widehat{x} \in \widehat{X}_m \cup \widehat{X}_w$ chooses a gamble γ from the given set $\Gamma(\widehat{x})$,
  - $\gamma(\cdot|\widehat{x})$ represents probability measure of a gamble.
  - Degenerate gamble $\gamma_0(\widehat{x}|\widehat{x}) = 1$ is always available.
  - Fair gambles: $\int x d\gamma(x|\widehat{x}) = \widehat{x}$.
- $\sigma_m(\widehat{x}_m)$ and $\sigma_w(\widehat{x}_w)$ represent gambling choices.

Matching Phase

- Gambles σ_m and σ_w induce μ_m and μ_w.
- Surplus function $s(x_m, x_w)$; singles produce zero.
- Matching market outcome
  - **Matching measure** μ describes the measure of matches.
  - **Payoff functions** $v_m : X_m \to \mathbb{R}_+$ and $v_w : X_w \to \mathbb{R}_+$.
- Stable outcome
  1. μ has marginals μ_m and μ_w.
  2. $v_m(x_m) + v_w(x_w) = s(x_m, x_w)$ if $(x_m, x_w) \in \text{supp}(\mu)$.
  3. $v_m(x_m) + v_w(x_w) \geq s(x_m, x_w)$ for any x_m and x_w.

Equilibrium

- Primitives of the model: $(\widehat{\mu}_m, \widehat{\mu}_w, \Gamma(\cdot), s)$.
- $(\sigma_m^*, \sigma_w^*, \mu_m^*, \mu_w^*, \mu^*, v_m^*, v_w^*)$ is an equilibrium if
  - Equilibrium strategies σ_m^* and σ_w^* maximize the agents' expected payoffs,
  - Equilibrium measures of characteristics μ_m^* and μ_w^* are induced by equilibrium strategies σ_m^* and σ_w^*, and
  - Equilibrium outcome (μ^*, v_m^*, v_w^*) is a stable outcome of equilibrium matching market (μ_m^*, μ_w^*).

Equilibrium Existence

- Construct a correspondence

$$\Phi : (v_m, v_w) \mapsto (\sigma_m, \sigma_w) \mapsto (\mu_m, \mu_w) \rightrightarrows (\mu, v'_m, v'_w).$$

- An equilibrium exists if $(v_m, v_w) = (v'_m, v'_w)$.
- By Glicksberg, an equilibrium exists if the set of stable payoff functions (v_m, v_w) is compact, convex, and non-empty valued, and Φ is upper-hemicontinuous, non-empty valued, convex-valued, and compact-valued.
  - Stable payoff functions are uniformly bounded and equicontinuous and use the Arzela-Asocli Theorem.
  - The map from (v_m, v_w) to (σ_m, σ_w) is continuous.

Stochastically Dominated Gambles

Proposition

Suppose that $s(x_m, x_w)$ is linear in x_m. Then, each man prefers a second-order stochastically dominated gamble.

Claim

In general, a person can prefer a second-order stochastically dominated investment gamble with lower expected matching characteristics. (This result helps to rationalize observed seemingly irrational/risk-loving career choice, for example, entrepreneurship).

Link between Stability and Competition

- x_m and x_w share the entire surplus,

$$v_m(x_m) = s(x_m, x_w) - v_w(x_w) \quad \text{if } (x_m, x_w) \in \text{supp}(\mu).$$

- x_m does not want to marry any woman other than x_w,

$$v_m(x_m) \geq s(x_m, x_w) - v_w(x_w) \quad \forall x_w \in \text{supp}(\mu_w).$$

- x_m marries woman $x_w(x_m)$ that gives him highest payoff,

$$\mathbf{x}_w(x_m) \in \text{argmax}_{x_w \in \text{supp}(\mu_w)}[s(x_m, x_w) - v_w(x_w)].$$

Competitive Rematching Effect

$$\mathbb{E}\Big[v_m\left(x_m\right)\Big] - v_m\left(\widehat{x}_m\right)$$

$$=$$

$$\mathbb{E}\Big[s\left(x_m, \mathbf{x}_w\left(x_m\right)\right) - v_w\left(\mathbf{x}_w\left(x_m\right)\right)\Big] - \Big[s\left(\widehat{x}_m, \widehat{x}_w\right) - v_w\left(\widehat{x}_w\right)\Big]$$
$$-\mathbb{E}\Big[s\left(x_m, \widehat{x}_w\right) - v_w\left(\widehat{x}_w\right)\Big] + \mathbb{E}\Big[s\left(x_m, \widehat{x}_w\right) - v_w\left(\widehat{x}_w\right)\Big]$$

$$=$$

$$\underbrace{\mathbb{E}\Big[s\left(x_m, \widehat{x}_w\right) \cancel{-v_w\left(\widehat{x}_w\right)}\Big] - \Big[s\left(\widehat{x}_m, \widehat{x}_w\right) \cancel{-v_w\left(\widehat{x}_w\right)}\Big]}_{\text{surplus contribution effect}}$$

$$+$$

$$\underbrace{\mathbb{E}\left\{\Big[s\left(x_m, \mathbf{x}_w(x_m)\right) - v_w\left(\mathbf{x}_w(x_m)\right)\Big] - \Big[s\left(x_m, \widehat{x}_w\right) - v_w\left(\widehat{x}_w\right)\Big]\right\}}_{\text{competitive rematching effect}\geq 0}$$

Competitive Rematching Effect under ITU

$$\mathbb{E}\left[v_m\left(x_m\right)\right] - v_m\left(\widehat{x}_m\right)$$

$$=$$

$$\mathbb{E}\phi\left(x_m, \mathbf{x}_w\left(x_m\right), v_w\left(\mathbf{x}_w\left(x_m\right)\right)\right) - \phi\left(\widehat{x}_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right)$$
$$-\mathbb{E}\phi\left(x_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right) + \mathbb{E}\phi\left(x_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right)$$

$$=$$

$$\underbrace{\mathbb{E}\phi\left(x_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right) - \phi\left(\widehat{x}_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right)}_{\text{surplus contribution effect}}$$

$$+$$

$$\underbrace{\mathbb{E}\left\{\phi\left(x_m, \mathbf{x}_w\left(x_m\right), v_w\left(\mathbf{x}_w\left(x_m\right)\right)\right) - \phi\left(x_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right)\right\}}_{\text{competitive rematching effect}\geq 0}$$

Relation to Becker et al. (2005, JPE)

- Becker et al. (2005, JPE) claim two indispensable factors that drive gambling in hedonic markets. Both factors are shown to be dispensable in two-sided gambling and matching.

  1. Complementarity between money and status.
  2. Fixed supply of status goods (one-sidedness).

- Another implication is that efficiency leads to inevitable inequality.

An Example with Two Equilibria

- Mass 1 of characteristics 2 men.
- Mass 1 of characteristics 2 women.
- Gambling options: 2 vs $\frac{1}{2} \circ 1 + \frac{1}{2} \circ 3$ (or equivalently in equilibrium, any fair gamble with realization between 1 and 3).
- Surplus $s(x_m, x_w) = x_m x_w$.

Two Equilibria

1. No-Gambling Equilibrium: No one gambles
   - Mass 1 of $(2, 2)$ matches.
   - $v^*(1) = 0$, $v^*(2) = 2$, $v^*(3) = 4$.
   - $SW^* = (1)(2)(2) = 4$.
2. Gambling Equilibrium: Everyone gambles
   - Mass 0.5 of $(1, 1)$ matches and mass 0.5 of $(3, 3)$ matches.
   - $v^*(1) = 0.5$, $v^*(2) = 1.5$, $v^*(3) = 4.5$.
   - $SW^* = (0.5)(3)(3) + (0.5)(1)(1) = 5$.

Problems

1. The no-gambling equilibrium is inefficient.
2. The gambling equilibrium creates inequality.
3. The government has no revenue.

Remedy 1: Tax on Matching Payoffs

A remedy: $[0,1)$ to 1; $[1,3)$ no tax; tax $2/3$ on $[3,\infty)$.

1. Eliminates the inefficient equilibrium
   - $v^\tau(1) = 1$, $v^\tau(2) = 2$, $v^\tau(3) = 3\frac{1}{3}$.
2. Reduces inequality
   - $v^\tau(1) = 1$, $v^\tau(2) = 2$, $v^\tau(3) = 3.5$.
3. Government generates positive tax revenue
   - $\tau = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot (-0.5) = \frac{1}{2}$.

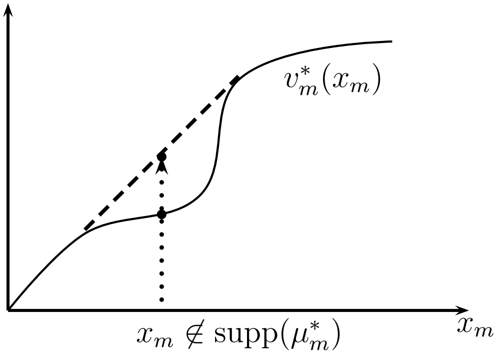
Remedy 2: Tax on Matching Types (Incomes)

A remedy: income 1 tax-free; tax income 2 at 15% to 1.7; tax income 3 at 16.66...% to 2.5.

1. Eliminates the inefficient equilibrium
   - $v_m^\tau(1) = 1 \times 1.7 - v_w^\tau(1.7)$, $v_m^\tau(2) = 1.7 \times 1.7 - v_w^\tau(1.7)$, $v_m^\tau(3) = 2.5 \times 1.7 - v_w^\tau(1.7)$.
2. Reduces inequality
   - $v_m^\tau(1) = 0.5$, $v^\tau(1.7) = 1.445$, $v^\tau(3) = 3.125$.
3. Government generates positive tax revenue
   - $\tau = \frac{1}{2} \cdot (0.5) + \frac{1}{2} \cdot (0.5) = 1$.

Concave Equilibrium Payoff Functions

The equilibrium payoff functions are weakly concave on equilibrium support and weakly convex outside the equilibrium support.



Conclusion

- People (men/women, college students, hedge fund managers) gamble due to matching concerns.
- Two-sided gambling could be socially efficient but cause inequality; could be equal but socially inefficient. (Carefully designed) taxation could eliminate inefficiency, mitigate inequality, and generate positive revenue.
- Explain gender differences in occupational choices and marital timing.

Gambling in Equilibrium Two-Sided Matching

A new idea

Gary S. Becker <gbecker@uchicago.edu> Sat, Feb 22, 2014 at 2:44 PM
To: Hanzhe Zhang <hanzhe@uchicago.edu>

Hanzhe,
Looked over your paper on gambling. Nicely done.
In my discussion in 301 of gambling, I often use a marriage example. Suppose a good and bad marriage, and by gambling you get the resources to go into a good marriage.The assumption I make is that the net utility from a bad marriage (net of all transfers to spouse, etc) is better when I have low incomes, but worse when I have high incomes. Then I would take a fair gamble; if I lose I get the bad marriage and if I win i get the good marriage. The shift from bad to good marriage makes the net utility function convex.

I believe there is a similarity of this discussion to what you do, but I like that you put it into an equilibrium two-sided matching framework. That is a significant advance over the literature.

I will read more carefully.
Gary Becker

THANK YOU!

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