Pre-Matching Gambles

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Abstract

This paper initiates the investigation of pre-matching gambles. Examples of pre-matching gambles include occupational choices before the marriage market, college major choices before the labor market, and financial portfolio management to attract future investors. I show that people take risky investments they would have not taken if not for their subsequent participation in competitive matching markets. A fundamental and unique feature of the competitive matching market, which I call the competitive rematching effect, drives pre-matching gambling. The paper then illustrates the inevitable relationship between social efficiency and inequality in this setting, and shows how progressive taxation in the matching market eliminates social inefficiency, reduces inequality, and generates government revenue.

Keywords: investment-and-matching, competitive rematching effect, efficiency and inequality, progressive taxation

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"The lottery of the law [profession] ... is very far from being a perfectly fair lottery; and that as well as many other liberal and honorable professions, is, in point of pecuniary gain, evidently under-recompensed. Those professions keep their level, however, with other occupations; and, notwithstanding these discouragements, all the most generous and liberal spirits are eager to crowd into them."

Adam Smith, Wealth of Nations (1776)

1 Introduction

Everybody gambles. Ordinary people like you and I visit casinos and buy lottery tickets. Sane, otherwise law-abiding citizens commit crimes and take chances to park illegally. College graduates pick career paths with similar expected lifetime earnings but very different income distributions. Young professionals quit their steady jobs to become entrepreneurs. Investment managers build risky portfolios. Countries choose to specialize in obscure, yet potentially rewarding industries.

Many reasons for gambling have been proposed. Smith (1776) suggests that social status motives and overconfidence drive these risky choices. Friedman and Savage (1948) postulate that utility functions exhibit convexity in some range. Rubin and Paul (1979) point out that people may steal, rob, or commit other risky crimes in order to reach subsistence level. Robson (1992) and Becker et al. (2005) reemphasize the importance of social status and introduce the market for "status goods."

I introduce a new reason for gambling - matching concerns. Many major decisions are made in matching markets: for example, the housing market, the labor market, and the marriage market. The competitive organization of matching markets can induce gambling. College students choose different majors that provide different bundles of human capital in order to meet future employers' needs in the labor market. Investment managers build risky portfolios because financial investment returns not only augment cash flow but also attract future investors. The unmarried accumulate and invest their wealth in risky ways in order to attract better mates in the future.

This paper is the first to study gambling behavior in an equilibrium two-sided matching market, to the best of my knowledge. Agents can pick lotteries to change their matching characteristics before they match and bargain their surplus. Investment gambles, matching characteristics, matching pattern, and the division of matching surplus are all endogenously determined. Studying these often overlooked risky investments in an equilibrium setting can lead to quite surprising implications about private and social risk-taking. This paper shows that the inherent competitive structure of the matching market encourages gambling.

The paper contains the following key results. First, the paper rationalizes risky investment choices before people enter the matching market. Second, the paper discovers a gambling-inducing factor inherent in a competitive matching market. Finally, the paper derives implications about the trade-off between efficiency and inequality in this setting.

Profitable pre-matching gambles are prevalent. Moderate risk aversion does not preclude agents from taking unfair, risky gambles. I first will show with motivating examples that an agent may strictly prefer to take an actuarially unfair gamble. More examples are given to show that the shape of the surplus function and the distributions of matching characteristics are not crucial drivers of gambling.

The competitive nature of the matching market encourages and induces pre-matching risk-taking. The competitive rematching effect always prompts the agent to gamble. A gamble does not only change an agent's contribution to surplus but also changes the partner he or she is matched with. Crucially, by reinterpreting the stability condition to a competitive condition, for every realization of the gamble, an agent is matched with the partner that gives him or her the highest attainable payoff. (Note that, without surplus supermodularity, the partner that helps attain the highest payoff is not necessarily the partner with the highest characteristics.) This competitive rematching effect is exclusive to the competitive matching market. A competitive consumption market alone is not enough, as a later example demonstrates.

Risk-taking, despite its usual negative connotations, is not necessarily undesirable in the current matching setting. Gambles can be beneficial to private as well as social welfare. I will show an example in which there are two equilibria: one efficient equilibrium in which everyone gambles and another inefficient equilibrium in which only one gambles. When multiple equilibria arise, carefully designed tax schemes can eliminate the inefficient equilibrium. I will show that a progressive tax scheme can effectively enhance social efficiency, reduce inequality, and generate positive tax revenue.

After a brief literature review, the rest of the paper proceeds as follows. Section 2 provides examples in which agents could find profitable pre-matching gambles. Section 3 describes the basic model. Section 4 highlights how the competitive nature of the matching market can drive pre-matching gambles. Section 5 discusses the implications for inequality, efficiency, and progressive taxation. Section 6 concludes.

Contributions to Literature

This paper is the first to investigate investments with uncertain returns, extending the previous investment-and-matching models in which investments yield deterministic returns (Cole et al., 2001; Iyigun and Walsh, 2007; Chiappori et al., 2009; Mailath et al., 2013; Dizdar, 2013; Mailath et al., 2015). Recently, Chade and Lindenlaub (2015) also considered pre-matching risky investments. In contrast to this current paper that focuses on the gambling mechanisms and the equilibrium properties, they focused on comparative statics when one side of the market gambles (gambling on both sides is discussed only to a limited extent). In particular, they focused on how aggregate macroeconomic risk, a factor not of this paper's interest, affects population's investment incentives.

The paper provides a new reason for gambling. Smith (1776) raises and discusses the issue. The primary consideration has been the additional social status wealth adds. Modern treatment begins with Friedman and Savage (1948). They argued that utility over money is convex on certain range, and people take gambles if their wealth falls in that range. Their explanation for the convexity is that people gamble with wealth because of social status considerations, as wealth gain raises not only consumption but also one's social status. Ray and Robson (2012) formalized these arguments in a dynamic setting. The current paper improves Becker et al. (2005) which investigated the gambling incentives when social status can be bought and traded in an explicit or implicit market. Their paper emphasizes that the complementarity between money and status for utility is key to prompt risk-taking behavior that results in unequal wage distributions, and the fixed supply of "status goods" is also key to prompt people to compete and gamble. This paper shows that surplus complementarity is no longer key in a two-sided hedonic market, and the fixed supply of "status goods" is also not important, because both sides can take gambles and the wealth distributions are endogenously determined as a result.

This paper also elaborates on the idea that the marriage market can generate gambling. Rubin and Paul (1979) and Robson (1996) both discuss incentives to gamble in order to obtain additional wives. Because of the discreteness of the number of wives a man can have, he would rather take unfair lotteries to obtain enough wealth and resources for an additional wife than to let some of resources to remain idle for inefficient use. Those papers rely on both assumptions of non-transferable utility and polygamy to generate gambling. In contrast, this current paper is in a transferable utility, one-to-one matching setting. In other words, the previous papers consider how *quantity* can affect a man's gambling incentive in a polygamous society, whereas the current paper considers how *quality* can induce similar gambling incentives in a monogamous society. In addition, the model herein offers a set of testable implications on differences in risk-taking behavior by gender and marital status.

Finally, the paper comments on the trade-off between efficiency and inequality, and offers insights on how a progressive taxation scheme can fix the problems. Rosen (1997) shows agents' willingness to gamble when higher wages enable one to move to a larger city with more abundant resources. From the literature that investigates voluntary and wealth redistribution, the paper provides a possible relationship between efficiency and inequality in a matching market. Rosen (1997) and Becker et al. (2005) investigate wealth redistribution as a result of gambling incentives. Bergstrom (1986) studies gambling and occupational choices.

2 Motivating Examples

Let me start with an example in which agents always want to take a risky investment before they enter a matching market.

Consider a simple continuum version of the standard Becker-Shapley-Shubik matching market. Namely, let there be masses 1 of men and women who have characteristics x_m, x_w uniformly distributed on $[0, 1]$. A man with characteristic x_m and a woman with characteristic x_w produce a surplus of $s(x_m, x_w) = x_m x_w$ when they match. Men and women in the matching market match frictionlessly and bargain to divide their surplus. Let $v_m(x_m)$ denote the payoff an x_m man gets, and $v_w(x_w)$ the payoff an x_w woman gets. In a stable outcome, when an x_m man and an x_w woman match, they divide their surplus between them: $v_m(x_m) + v_w(x_w) = x_m x_w$. In addition, for any pair of an x_m man and an x_w woman, stability requires that no pair of agents can generate more surplus than in their current match: $v_m(x_m) + v_w(x_w) \geq x_m x_w$.

The stable outcome of this matching market is simple to characterize. The result is well known: supermodularity of the surplus function results in positive assortative matching, every x_m man matches with an $x_w = x_m$ woman. Every x_m man and his wife of the same characteristic $x_w = x_m$ produce a surplus of x_m^2. Each x_m man gets $x_m^2/2$ and each x_w woman gest x_w^2. That is, $v_m(x_m) = x_m^2/2$ and $v_w(x_m) = x_w^2/2$.

Clearly, the payoff functions are convex in the agents' characteristics. A man of characteristic 0.5 has a payoff of $0.5^2/2 = 0.125$. Suppose that he can take a risky investment before he enters the matching market to change his matching characteristic. Namely, he can take an investment that makes him either a man of characteristic 0.4 with probability 1/2 or a man of characteristic 0.6 with probability 1/2. This investment keeps his expected characteristic to be 0.5. When he becomes of characteristic 0.4, his partner will be a woman of characteristic 0.4. When he becomes of characteristic 0.6, he will match with a woman of characteristic 0.6. With probability one half, his payoff is $0.4^2/2 = 0.08$, and with the other half probability, his payoff is $0.6^2/2 = 0.18$. The expected payoff is 0.13, which is higher than 0.125 (the payoff without gambling). Therefore, he prefers such a risky investment to the safe investment. In fact, among all the risky investments that have the same expected characteristic of 0.5, he prefers to take the investment that has the highest risk! When he takes the gamble that makes him of characteristic 1 with half probability and characteristic 0 with half probability, his expected payoff is $\frac{1}{2}(1^2/2) + \frac{1}{2}(0^2/2) = 0.25$, double the payoff if he does not gamble at all!

Moreover, the gain from gambling is so high that he may prefer an investment that is not only riskier but also yields a lower expected characteristic. For example, a man of characteristic 0.5 prefers to take a gamble that yields characteristic 1 with probability p and characteristic 0 with probability $1 - p$ as long as $p > 1/4$. The gamble only has an expected characteristic of p; the gamble is unfair if $p < 1/2$. Moreover, even if the man is risk-averse in payoffs, he can still prefer to take the second-order stochastically dominated gamble as long as he is not too risk-averse.

Recognize that if such a matching market does not exist and the man's partner is fixed to have characteristic x_w, then all the different mean-spreading investments (0.5, $\frac{1}{2}\circ 0.4 + \frac{1}{2}\circ 0.6$, $\frac{1}{2}\circ 0 + \frac{1}{2}\circ 1$, etc.) yield him exactly the same expected payoff ($\mathbb{E}v_m(x_m) = \mathbb{E}[x_m x_w - v_w(x_w)] = 0.5x_w - v_w(x_w)$). Without the matching market, it is never optimal for a man to take a second-order stochastically dominated gamble. The stability of the matching market plays a quintessential role in inducing gambling. Intuitively speaking, the stable matching market provides the gambling incentive, because for different realizations of the gambles, the man matches with a different partner. A gamble changes the man's matching characteristic. More importantly, it changes the partner the man is matched with in a systematically competitive way. Stability of the matching market induces such a gambling incentive.

The possible conjecture that supermodularity of the surplus function and the resulting positive-assortative matching, as in the example above, plays an important role in driving the gambling incentive is not entirely correct. Even if the surplus function is completely submodular, we would find the same pre-matching gambling incentives, as the example below demonstrates.

Example 1 (Gambling when surplus function is submodular)**.** Suppose that the surplus function $s(x_m, x_w) = x_m + x_w - x_m x_w$ is submodular in x_m and x_w. The stable matching is negative assortative; that is, a man of x_m matches with a woman of $x_w = 1 - x_m$. The payoffs satisfy for all $x_m \in [0, 1]$, $v_m(x_m) + v_w(1 - x_m) = 1 - x_m + x_m^2$. $v_m(x_m) = \max_{x_w}[s(x_m, x_w) - v_w(x_w)]$, so by envelope theorem, $v_m'(x_m) = \partial s(x_m, x_w(x_m))/\partial x_m = 1 - x_w(x_m) = x_m$, where $x_w(x_m)$ represents the partner an x_m man is matched to. Therefore, $v_m(x_m) = x_m^2/2 + 1/4$.

In addition, another possible conjecture that surplus function of homogeneous of degree

$r > 1$ plays an important role in driving the result is also not entirely correct. Gambling incentives exist even when the surplus function is homogeneous of degree $r < 1$, as the following example demonstrates.

Example 2 (Gambling when surplus function is homogeneous of degree $r < 1$)**.** Suppose the surplus function is $s(x_m, x_w) = \sqrt{x_m x_w}$ and the distributions of men's and women's characteristics are different: $\widehat{F}_m(x_m) = x_m^2$ and $\widehat{F}_w(x_w) = x_w$. In the stable outcome, each x_m man matches with a $x_w = x_m^2$ woman, and the payoffs are $v_m(x_m) = \frac{1}{3}x_m^{\frac{3}{2}}$ and $v_w(x_w) = \frac{2}{3}x_w^{\frac{3}{4}}$. Since the payoff function for men is convex, men want to gamble, even take extreme gambles, although the surplus function is homogenous of degree 1. Even if the surplus function is homogeneous of degree $r < 1$, $s(x_m, x_w) = x_m^{r/2} x_w^{r/2}$, and the distributions of men's and women's characteristics are $\widehat{F}_m(x_m) = x_m^2$ and $\widehat{F}_w(x_w) = x_w$. Men's payoff function is $v_m(x_m) = \frac{2}{2+r}x^{1+\frac{r}{2}}$, which is always convex when $r > 0$. For certain distributions, both men and women have incentives to gamble.

In fact, the gambling incentive exists not only independent of the surplus function, but also absent of the particular distributions of the matching characteristics in the market, masses of men and women in the market, as well as dimensionality of the characteristics.

The stability condition is the central driving force of the gambling incentive. Intuitively, this is best seen from a reinterpretation of the stability condition. Stability guarantees that no pair of agents wants to deviate from their current match and form a new match. For any individual agent, *stability guarantees that each agent in the market is matched with the partner that gives him or her the highest personal payoff*, although the partner may not be ranked the best by sheer personal characteristic. Although each agent is free to give up more personal gain to match with a partner who has a higher characteristic, it is not in his or her best interest to do so.

The equivalent mathematical formulation of the economic argument in the previous paragraph is as follows: stability dictates that a particular x_m^* man who matches with an x_w^* woman has payoff $v_m(x_m^*) = s(x_m^*, x_w^*) - v_w(x_w^*)$. If he matches with any other $x_w \neq x_w^*$ woman and pays her the competitive payoff $v_w(x_w)$, his payoff would be $s(x_m^*, x_w) - v_w(x_w)$. By the stability condition, for any x_m and x_w, $v_m(x_m) + v_w(x_w) \geq s(x_m, x_w)$, $v_m(x_m^*) \geq s(x_m^*, x_w) - v_w(x_w)$. The argument holds for any x_m^* and for any distribution of men and women.

Let's see how this reinterpretation of the stability condition helps us understand the gambling incentive when the surplus is $x_m x_w$, independent of the distribution of men and women. An x_m^* man matches with an x_w^* woman (notice that positive assortative matching resulting from supermodularity does not play a role), and they divide up the surplus,

$v_m(x_m^*) = x_m^* x_w^* - v_w(x_w^*)$. Suppose that the man takes a gamble that makes $x_m^* - \epsilon$ with probability 1/2 and $x_m^* + \epsilon$ with probability 1/2. After he realizes the outcome from the gamble, he will get a new partner according to the new outcome. No matter who the new match is, we know by stability that he is guaranteed a payoff $x_m x_w^* - v_w(x_w^*)$ by matching with x_w^*. That is, for any realization of x_m, $v_m(x_m) \geq x_m x_w^* - v_w(x_w^*)$. The expected payoff of the gamble is thus bigger than $\frac{1}{2}[(x_m - \epsilon)x_w^* - v_w(x_w^*)] + \frac{1}{2}[(x_m + \epsilon)x_w^* - v_w(x_w^*)] = x_m x_w^* - v_w(x_w^*)$, which is exactly the payoff $v_m(x_m^*)$ without gambling. As long as an $x_m^* + \epsilon$ or $x_m^* - \epsilon$ man does not match with x_w^*, gambling yields strictly bigger expected utility than not gambling, if women's characteristics are sufficiently heterogeneous.

3 Model

Let me set up the general model and define the equilibrium of the model in this section. There is a continuum of men and women with innate characteristics $\widehat{x}_m \in \widehat{X}_m \subset \mathbb{R}^{N_m}$ and $\widehat{x}_w \in \widehat{X}_w \subset \mathbb{R}^{N_w}$, which are possibly multidimensional. $\widehat{\mu}_m$ and $\widehat{\mu}_w$ respectively describe the measures of the innate characteristics. The mass of men, $\widehat{\mu}_m(\widehat{X}_m)$, and that of women, $\widehat{\mu}_w(\widehat{X}_w)$, are not necessarily equal. All agents are risk-neutral and derive utilities from the payoffs in the matching market.[1]

3.1 Gambling Phase

Each agent of innate characteristic $\widehat{x}$ can pick a gamble $\gamma(\cdot|\widehat{x}) \in \Gamma(\widehat{x})$ that alters his or her innate characteristic $\widehat{x}$, where γ represents the probability measure of realized matching characteristics. Assume the degenerate investment gamble that keeps the innate characteristic (i.e. $\gamma_0(\widehat{x}|\widehat{x}) = 1$) is always feasible. In addition, for each $\widehat{x}$, the set of feasible investments $\Gamma(\widehat{x})$ is compact. All gambles have the same expected characteristic: $\int x d\gamma(x|\widehat{x}) = \widehat{x}$. Let $\sigma_m : \widehat{x}_m \mapsto \gamma \in \Gamma(\widehat{x}_m)$ denote men's investment strategy and $\sigma_w : \widehat{x}_w \mapsto \gamma \in \Gamma(\widehat{x}_w)$ denote women's investment strategy. Let σ_m and σ_w be measurable. Let X_m and X_w denote the full support of possible realized characteristics. Since keeping the innate characteristic is always feasible, $X_m \supseteq \widehat{X}_m$ and $X_w \supseteq \widehat{X}_w$. After the agents choose the investments and before they enter the matching market, their matching characteristics x_m and x_w are realized.

[1] We name the two sides in the model men and women. Obviously, agents on the two sides of the matching market can also be workers and firms, consumers and goods, students and schools, or buyers and sellers.

3.2 Matching Phase

Let μ_m and μ_w represent the measures of men's and women's realized characteristics after they make their investment decisions. Let $s(x_m, x_w)$ denote the surplus produced by a match between an x_m man and an x_w woman. Suppose that remaining single yields zero surplus. Assume that s is continuous in two arguments. In the matching market (μ_m, μ_w), men and women frictionlessly match and bargain. They divide their surplus in a perfectly transferable way.

A stable outcome of the matching market consists of a matching measure denoted by μ on $X_m \times X_w$ and the payoff functions $v_m : X_m \to \mathbb{R}_+$ and $v_w : X_w \to \mathbb{R}_+$.

$\mu(\widetilde{X}_m, \widetilde{X}_w)$ describes the measure of matches between $x_m \in \widetilde{X}_m$ and $x_w \in \widetilde{X}_w$, and $v_m(x_m)$ and $v_w(x_w)$ describe the payoff an x_m man and an x_w woman get, respectively. The following conditions are satisfied: (i) μ has marginals μ_m and μ_w, (ii) $v_m(x_m) + v_w(x_w) = s(x_m, x_w)$ when $(x_m, x_w) \in \text{supp}(\mu)$ where supp represents the support of the measure, (iii) $v_m(x_m) + v_w(x_w) \geq s(x_m, x_w)$ for any $(x_m, x_w) \in \text{supp}(\mu_m) \times \text{supp}(\mu_w)$, and (iv) for any $x_m \in X_m$ and $x_w \in X_w$ outside of the supports $\text{supp}(X_m)$ and $\text{supp}(X_w)$, the payoff functions are defined as $v_m(x_m) = \max_{x_w \in \text{supp}(X_w)}[s(x_m, x_w) - v_w(x_w)]$ and $v_w(x_w) = \max_{x_m \in \text{supp}(X_m)}[s(x_m, x_w) - v_m(x_m)]$.

Condition (i) is the feasibility condition for the matching. Conditions (ii) and (iii) are the conditions that any matched pair splits their surplus, and that two agents not matched with each other cannot deviate and form a pair to have both of their payoffs strictly improve. Condition (iv) deals with defining the payoff of the characteristics outside of the post-gambling characteristics support. The payoff functions need to be well-defined on the entire support in order for the agents to make gambling decisions.

3.3 Equilibrium

The primitives of the model consist of the measures of innate characteristics $\widehat{\mu}_m$ and $\widehat{\mu}_w$, the feasible investment set $\Gamma(\cdot)$, and the surplus function s. An equilibrium

$$(\sigma_m^*, \sigma_w^*, \mu_m^*, \mu_w^*, \mu^*, v_m^*, v_w^*)$$

consists of equilibrium investment strategies σ_m^* and σ_w^*, equilibrium measures μ_m^* and μ_w^* of characteristics, and equilibrium matching outcome (μ^*, v_m^*, v_w^*) such that: (i) equilibrium strategies σ_m^* and σ_w^* maximize the agents' expected payoffs, (ii) equilibrium measures of characteristics μ_m^* and μ_w^* are induced by equilibrium strategies σ_m^* and σ_w^*, and (iii) equilibrium matching market outcome (μ^*, v_m^*, v_w^*) is a stable outcome of equilibrium matching market (μ_m^*, μ_w^*). An equilibrium exists by Zhang (2015).

4 Competitive Rematching Effect

4.1 Dominant Extreme Gambles Under Linear Surplus

When the surplus function is linear in the matching characteristics, it is a weakly dominant strategy to pick the extreme lotteries that realize either the highest possible characteristic or the lowest possible characteristic. It follows that there exists an equilibrium in which every man and every woman chooses the extreme lottery when the surplus function is bilinear.

Definition 1. An **extreme lottery** $\gamma(\cdot|\widehat{x})$ for an agent of innate characteristic $\widehat{x}$ is to realize only the highest or the lowest possible characteristic. The gamble yields $\overline{x}$ with probability p and $\underline{x}$ with probability $1-p$ so that $p \cdot \overline{x} + (1-p) \cdot \underline{x} = \widehat{x}$.

Note that when x is multidimensional, p is a vector.

Proposition 1. *If the surplus function $s(x_m, x_w)$ is linear in x_m, every man's unique weakly dominant strategy is the extreme lottery.*

First, when surplus is linear in men's characteristic, every man prefers any gamble to the degenerate gamble.

Lemma 1. *If $s(x_m, x_w)$ is linear in x_m, every man prefers a gamble to any second-order stochastically dominant gamble that generates the same expected characteristics.*

The result is independent of the distributions of characteristics, μ_m and μ_w. The expected payoff of taking the degenerate gamble $\gamma_0(\cdot|\widehat{x})$ for an $\widehat{x}_m$ man is $u_m(\gamma_0|\widehat{x}) = v_m(\widehat{x})$, and the expected utility of taking any other fair gamble $\gamma(\cdot|\widehat{x})$ is $u_m(\gamma|\widehat{x}_m) = \int v_m(x_m) d\gamma(x_m|\widehat{x}_m)$. Suppose $\widehat{x}_m$ is matched with an x_w^* woman, then they divide up the surplus:

$$v_m(\widehat{x}_m) = s(\widehat{x}_m, x_w^*) - v_w(x_w^*).$$

By stability, for any $x_m \neq \widehat{x}_m$,

$$v_m(x_m) \geq s(x_m, x_w^*) - v_w(x_w^*),$$

that is, any other man's payoff cannot be worse off than matching with the x_w^* woman. Plugging in the inequality, the expected utility from taking gamble γ is

$$u_m(\gamma|\widehat{x}_m) = \int v_m(x_m) d\gamma(x_m|\widehat{x}_m) \geq \int [s(x_m, x_w^*) - v_w(x_w^*)] d\gamma(x_m|\widehat{x}_m)$$

which is $\int s(x_m, x_w) d\gamma(x_m|\widehat{x}_m) - v_w(x_w)$. Since s is linear in x_m,

$$\int s(x_m, x_w) d\gamma(x_m|\widehat{x}_m) = s\left(\int x_m d\gamma(x_m|\widehat{x}_m), x_w\right).$$

Because $\gamma(x_m|\widehat{x}_m)$ is fair, $\int x_m d\gamma(x_m|\widehat{x}_m) = \widehat{x}_m$. Hence $u_m(\gamma|\widehat{x}_m) \geq s(\widehat{x}_m, x_w^*) - v_w(x_w^*) = v_m(\widehat{x}_m) = u_m(\gamma_0|\widehat{x}_m)$.

To show Proposition 1 generally, that a second-order stochastically dominated fair gamble γ is always preferred to a second-order stochastically dominant fair gamble γ', recognize that the second-order stochastically dominated gamble γ can be decomposed into the second-order stochastically dominant gamble γ' and another non-degenerate gamble. As the proof of Lemma 1 demonstrates, a gamble is always preferred to no gamble. Regardless of the characteristic, a man always prefers to have taken an even riskier gamble! The detailed proof can be seen in the Appendix.

Proof of Proposition 1. Consider a second-order stochastically dominated gamble γ and a second-order stochastically dominant gamble γ'. Let x_γ denote the random variable of the dominated gamble and $x_{\gamma'}$ the random variable of the dominant gamble. There exists a random variable z such that $x_\gamma = x_{\gamma'} + z$, $\mathbb{E}[z|x_{\gamma'}] = 0$. That is, x_γ can be obtained by adding mean-zero noise to $x_{\gamma'}$. Let $x_w(x_m)$ denote the partner an x_m man is matched with (if the match is not pure so that a man can be matched to women of different characteristics with positive probability, randomly pick any one of the women to match). For any $\widehat{x}_m$,

$$v_m(\widehat{x}_m) = s(\widehat{x}_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m)).$$

And for any $x_m \neq \widehat{x}_m$,

$$v_m(x_m) \geq s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m)).$$

The inequality, linearity of the surplus function, and the decomposition of x_γ into $x_{\gamma'}$ and z are used in the following derivation,

$$\begin{aligned}
u_m(\gamma|\widehat{x}_m) &= \int v_m(x_m) d\gamma'(x_m|\widehat{x}_m) \\
&= \int [s(x_m, x_w(x_m)) - v_m(x_w(x_m))] d\gamma'(x_m|\widehat{x}_m) \\
&= \int \left[\int s(x_m + z, x_w(x_m)) d\gamma_z(z) - v_m(x_w(x_m))\right] d\gamma'(x_m|\widehat{x}_m) \\
&= \int \int [s(x_m + z, x_w(x_m)) - v_m(x_w(x_m))] d\gamma_z(z) d\gamma'(x_m|\widehat{x}_m)
\end{aligned}$$

$$
\begin{aligned}
&\leq \int\int v_m(x_m+z)d\gamma_z(z)d\gamma'(x_m|\widehat{x}_m) \\
&\leq \int v_m(x_m)d\gamma(x_m|\widehat{x}_m) \\
&= u_m(\gamma'|\widehat{x}_m)
\end{aligned}
$$

□

Proposition 2. *Suppose the surplus function $s(x, y)$ is bilinear. There exists an equilibrium in which every agent takes the extreme gamble.*

The equilibrium in which everyone takes the extreme gamble is not necessarily the unique equilibrium. Section 5 presents an example with multiple equilibria. Note that bilinearity does not imply supermodularity of the surplus function. Whereas $s(x_m, x_w) = x_m x_w$ is bilinear and strictly supermodular, $s(x_m, x_w) = x_m + x_w - x_m x_w$ is bilinear and strictly submodular. The dominance of extreme gamble holds when the surplus function is bilinear, even if it is not supermodular.

4.2 Competitive Rematching Effect

The motivating example in Section 2 and the results that extreme gambles are the unique dominant strategies under linear surplus suggest that the stable payoff functions are inherently convex and that the stable and competitive organization of the assignment market itself contributes to the payoff convexity. I will show below that an inherent competitive rematching effect always generates convexity in the stable payoff functions and thus induces pre-matching gambling. Notably, this gamble-inducing effect arises solely as a byproduct of the stability conditions, and does not depend on any other structural assumption.

In essence, the stable assignment, in particular the rematching based on realized characteristics, provides an implicit but persistent benefit to gambling. In any stable outcome, a man cannot do strictly better by matching with a partner different from the one he is matched with and paying the new partner at least her competitive payoff. Every man is matched with the partner that maximizes his marital payoff. Every woman is matched with the partner that maximizes her marital payoff.

When a man gambles, he will match with a different partner of higher or lower characteristic depending on the realization of the gamble. This partner guarantees him a higher stable payoff than he would get otherwise with the partner he's matched with if he does not gamble. This competitive rematching effect always brings an extra benefit to the agent who gambles. The competitive rematching based on realized characteristic is treated as a benefit to the agent who realizes a higher characteristic and an insurance if the realized characteristic is lower than before gambling.

Let me formally demonstrate the gamble-inducing competitive rematching effect. Consider a matching market induced by the population strategies. Take any man x_m^*. Recall the stability conditions that stable matching and payoffs satisfy. For any woman $x_w \in \text{supp}(g_w)$,

$$v_m(x_m^*) \geq s(x_m^*, x_w) - v_w(x_w).$$

If an x_m^* man and an $x_w(x_m^*)$ woman are matched, i.e. $(x_m^*, x_w(x_m^*)) \in \text{supp}(\mu)$, then

$$v_m(x_m^*) = s(x_m, x_w(x_m^*)) - v_w(x_w(x_m^*)).$$

$s(x_m, x_w) - v(x_w)$ represents an x_m man's payoff if he marries an x_w woman and pays her the competitive market value $v_w(x_w)$. In the stable outcome, x_m^* is matched with $x_w(x_m^*)$ only if for all x_w,

$$s(x_m^*, x_w(x_m^*)) - v_w(x_w(x_m^*)) \geq s(x_m^*, x_w) - v_w(x_w).$$

That is, in any stable outcome, *every man marries the woman that gives him the highest possible private payoff*, i.e.,

$$v_m(x_m^*) = \sup_{x_w \in \text{supp}(\mu_w)} [s(x_m^*, x_w) - v_w(x_w)].$$

The same statement can be said of women: in any stable outcome, *every woman marries the man that gives her the highest possible private payoff*, i.e.,

$$v_w(x_w) = \sup_{x_m \in \text{supp}(\mu_m)} [s(x_m, x_w) - v_m(x_m)].$$

After a man of innate characteristic $\widehat{x}_m$ takes a gamble and realizes a characteristic x_m, his partner in the market changes, and the payoff he or she gets by matching with the new partner exceeds the payoff he gets by matching with the original pre-gamble partner $x_w(\widehat{x}_m)$, i.e.,

$$v_m(x_m) = s(x_m, x_w(x_m)) - v_w(x_w(x_m)) \geq s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m)). \tag{1}$$

Compare a feasible fair lottery $\gamma(\cdot|\widehat{x}_m)$ for an $\widehat{x}_m$ man to his degenerate gamble γ_0. The difference between the expected payoffs from the two decisions is

$$u_m(\gamma|\widehat{x}_m) - u_m(\gamma_0|\widehat{x}_m) = \mathbb{E}_\gamma[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(\widehat{x}_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))].$$

Subtract and add the same term $\mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m) - v_w(x_w(\widehat{x}_m))]$, the hypothetical expected payoff $\widehat{x}_m$ would receive by taking gamble γ, matching with woman $x_w(\widehat{x}_m)$, and transferring $v_w(x_w(\widehat{x}_m))$ to the partner. The expected payoff difference $u_m(\gamma|\widehat{x}_m) - u_m(\gamma_0|\widehat{x}_m)$ is rewritten as

$$\begin{gathered}\mathbb{E}_\gamma[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - \mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))] \\ + \\ \mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))] - [s(\widehat{x}_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]\end{gathered}$$

Combine the first two terms and the last two terms respectively, the expected payoff difference is expressed as the sum of two effects,

$$\begin{gathered}\underbrace{\mathbb{E}_\gamma\left\{[s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))] - [s(\widehat{x}_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]\right\}}_{\text{surplus contribution effect}} \\ + \\ \underbrace{\mathbb{E}_\gamma\left\{[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]\right\}}_{\text{competitive rematching effect}}\end{gathered}$$

The first combined term represents the difference between two expected payoffs: (1) the expected payoff $\widehat{x}$ gets by taking the gamble γ but matching with the partner that $x_w(\widehat{x}_m)$ he would have matched with without gambling, and (2) the (expected) payoff of $\widehat{x}$ for not gambling and always matching with $x_w(\widehat{x}_m)$. Since the wife always gets her payoff $v_w(x_w(\widehat{x}_m))$ regardless of the realization of the characteristic, the first combined term can be simplified to

$$\underbrace{\mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m))] - s(\widehat{x}_m, x_w(\widehat{x}_m))}_{\text{surplus contribution effect}}.$$

If the surplus function is convex in x_m, i.e. a man's marginal surplus contribution increases as his characteristic increases, then the effect is positive. If the surplus function is concave, or the marginal surplus decreases as man's characteristic increases, then the effect is negative. I call this term the **surplus contribution effect** since its sign depends on the slope of marginal surplus function and the convexity of the surplus function. If the surplus function is linear in x, then this term is always zero and this effect does not affect people's gambling incentives at all. Therefore, the previous results, namely Lemma 1 and Proposition 1, on beneficial gambling under bilinear surplus function, must be driven by the second effect.

The second combined term represents the expected payoff difference from optimal partner rematching based on different gambling realizations. For any realized x_m,

$$[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]$$

represents the difference between (1) the maximal payoff man x_m can get by matching with $x_w(x_m)$ and the possibly non-optimal payoff x_m gets by matching with $x_w(\widehat{x}_m)$. By Equation (1) and how $s(x_m, x_w(x_m)) - v_w(x_w(x_m))$ is the maximal payoff for x_m, the payoff difference is non-negative for any realized x_m. Since the payoff difference is non-negative for any realization, the expected payoff difference over all possible realizations is always non-negative. I call the second term the **competitive rematching effect** because the payoff gain comes from competitively rematching the agent to an optimally chosen partner. This rematching benefit gives a persistent reason for agents to gamble.

Note that the competitive rematching effect does not depend on any assumption about the shape of the surplus function or the distributions of the matching characteristics but comes solely from the stability conditions. The stability condition is reinterpreted to be a competitive condition such that the stability guarantees the agents a competitive maximal payoff from their matched partners.

To emphasize that stability induces gambles regardless of the shape of the surplus function, let's take a price-theoretic approach to see why in particular the surplus supermodularity assumption is dispensable. Consider a matching market where the matching characteristics are one-dimensional and are represented by distributions of characteristics. Suppose that the mass distributions F_m and F_w are strictly increasing and twice differentiable on full supports $X_m \in \mathbb{R}_+$ and $X_w \in \mathbb{R}_+$ and that s is strictly supermodular and twice differentiable. The stable matching is positive assortative: $x_w(x_m) = F_w^{-1}(F_m(x_m))$ is strictly increasing and bijective. Moreover, v_m and v_w are differentiable. An x_m man's payoff is the surplus x_m and $x_w(x_m)$ produce together net the payoff of $x_w(x_m)$ woman,

$$v_m(x_m) = s(x_m, x_w(x_m)) - v_w(x_w(x_m)).$$

Each man is paired with the woman that maximizes his payoff, so we have the first order condition

$$s_2(x_m, x_w(x_m)) - v_w'(x_w(x_m)) = 0. \tag{2}$$

Let's examine the convexity of the continuous and twice differentiable payoff function v_m. Differentiate $v_m(x_m)$ with respect to x_m, by the Envelope Theorem,

$$v_m'(x_m) = s_1(x_m, x_w(x_m)) + [s_2(x_m, x_w(x_m)) - v_w'(x_w(x_m))]x_w'(x_m).$$

By the First Order Condition (2), the second term is zero, and $v'_m(x)$ is simply the marginal surplus of x_m given the optimal partner $x_w(x_m)$, i.e.

$$v'_m(x_m) = s_1(x_m, x_w(x_m)),$$

a standard and widely known result in matching literature. Differentiate $v'_m(x_m)$ with respect to x_m,

$$v''_m(x_m) = \underbrace{s_{11}(x_m, x_w(x_m))}_{\text{surplus contribution effect}} + \underbrace{s_{12}(x_m, x_w(x_m))x'_w(x_m)}_{\text{stable rematching effect}}.$$

The two terms correspond to the two effects described above - the surplus contribution effect and the competitive rematching effect. The second term, the competitive rematching, is unambiguously weakly positive. When the surplus function is strictly supermodular, $s_{12} > 0$, the stable matching is positive assortative so $x'_w(x_m) > 0$, so the effect is positive.

When the surplus is strictly supermodular, it is straightforward to understand gambling incentives to improve expected equilibrium marginal surplus. With a supermodular surplus function, an agent's own marginal surplus increases in partner's characteristic, so the agent has incentive to take a fair gamble to be matched with a better partner and enjoys a higher marginal surplus. The gambling incentives in the one-sided hedonic market in Rosen (1997) and Becker et al. (2005) for example crucially depend on the assumption of complementarity.

Although the gambling incentives in the matching market can be justified in the same way when the surplus is supermodular, that does not imply that gambling incentives crucially depend on the supermodularity assumption. Take the extreme opposite case that the surplus function is submodular, $s_{12} < 0$ (e.g. $s(x_m, x_w) = x_m + x_w - x_m x_w$ for $x_m, x_w \in [0, 1]$). An agent's marginal surplus *decreases* as the partner's characteristic increases. However, in the matching market, the agent is matched with a partner based on the realized characteristic. A higher realized characteristic results in a partner with a *lower* characteristic when the surplus is submodular. Consequently, when an agent realizes a higher characteristic, his competitively matched partner has a lower characteristic than if he does not take the gamble; when an agent realizes a lower characteristic, his stably assigned partner has a higher characteristic. Isolating the competitive rematching effect, $s_{12} < 0$ and $x'_w(x_m) < 0$ imply $s_{12}(x_m, x_w(x_m))x'_w(x_m) > 0$.

We see from the elaborations above that the competitive rematching effect always contributes to the convexity of the payoff function regardless of the underlying surplus function. Suppose the surplus function is strictly supermodular for certain pairs (x_m, x_w) and strictly submodular for other pairs (x_m, x_w). When $s_{12} > 0$, the stable matching is locally positive assortative. On the other hand when $s_{12} < 0$, the stable matching is locally negative assortative. Therefore, $s_{12}(x_m, x_w(x_m))x'_w(x_m) \geq 0$ for all pairs of (x_m, x_w). As long as the surplus contribution effect is not significantly negative, the competitive rematching effect always encourages gambling behavior.

An important condition that guarantees strict gambling incentive for agents and how much gambling the agents do is the degree of diversity on the opposite side of the market. Take the extreme case that all women are born identical and do not gamble. Then $x_w(x_m) = x_w(x'_m)\ \forall x_m \neq x'_m$. The competitive rematching effect disappears completely, because it crucially depends on the positive probability that men rematch to different partners after gambling. Conversely, if the other side of the two-sided market is diverse, then gambling becomes more attractive. Mathematically,

$$v_m(x_m) = [s(x_m, x_w(x_m)) - v_w(x_w(x_m))] = \sup_{x_w \in \text{supp}(\mu_w)} [s(x_m, x_w) - v_w(x_w)].$$

As the diversity of women's matching characteristics increases, $\text{supp}(\mu_w)$ expands, any man's optimal payoff weakly increases. The payoff gain due to rematching increases without affecting the magnitude of the surplus contribution effect. The degree of diversity is important in guaranteeing uniqueness of the equilibrium with socially efficient investments in Cole et al. (2001). It also will play a crucial role in this model. In Section 5, I show an example with homogeneous agents on both sides of the market and multiple equilibria.

The dissection into the surplus contribution effect and the competitive rematching effect helps us understand our results in the previous subsection with a bilinear surplus function. When the surplus function is linear in men's characteristic, the surplus contribution effect disappears for men. When the surplus function is bilinear (linear in both men's and women's characteristics), the surplus contribution effect disappears for both men and women. Convexity of the stable payoff function hinges on the competitive rematching effect. In the special case of a bilinear surplus function, $s_{11} = s_{22} = 0$, the surplus contribution effect is mute and the stable payoff functions exhibit weak convexity generally and strict convexity when the agents are heterogeneous enough.

The gamble-inducing competitive rematching effect is unique to the matching market. The competitive rematching effect is similar to a substitution effect. The competitive rematching rematches a man to a woman when his characteristics change, whereas when the total income changes, there is substitution between goods consumed. But they also differ in a crucial way. The competitive rematching effect drives gambling, but the substitution effect in general does not. The matching market is special in the following sense. Consider a competitively organized market. A vector of goods $\{1, \cdots, N\}$ is available, and a bundle is denoted by $\mathbf{x} = (x_1, \cdots, x_N) \in \mathbf{X}$. Suppose the supply of goods is fixed. Every person is endowed with (possibly different) wage earnings $w_i \in \mathbb{R}_+$ and (possibly different, "well-behaved") utility function $u_i : \mathbf{X} \to \mathbb{R}_+$. They are price takers and denote the vector of prices by $\mathbf{p}$.

The utility a person i derives when he has income w is the maximal utility he can derive by consuming the optimal bundle of goods,

$$u(w) \equiv u(\mathbf{x}) \quad \text{s.t. } \mathbf{p}{\cdot}\mathbf{x}{\le}w$$

Let $\mathbf{x}(w)$ denote the optimal bundle of goods when a person has income w.

Consider gambling before making a purchasing decision in the competitive consumption market. Suppose a person starts with income $\widehat{w}$ and can take a fair gamble on the income. The person purchases the goods after gambling. Then the utility difference between gambling and not gambling is

$$\mathbb{E}[u(w)] - u(\widehat{w}),$$

and express in terms of the explicit utility function,

$$\mathbb{E}[u(w)] - u(\widehat{w}) = \mathbb{E}[u(\mathbf{x}(w))] - u(\mathbf{x}(\widehat{w})).$$

Add and subtract the term $\mathbb{E}\left[u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right)\right]$, i.e. the expected utility if the agent simply consumes the feasible bundle given any income w by shifting proportionally with respect to his income $\widehat{x}$, the difference becomes

$$\mathbb{E}[u(\mathbf{x}(w))] - \mathbb{E}\left[u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right)\right] + \mathbb{E}\left[u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right)\right] - u(\mathbf{x}(\widehat{w})).$$

It combines to have two terms,

$$\underbrace{\mathbb{E}\left[u(\mathbf{x}(w)) - u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right)\right]}_{\text{income effect}} + \underbrace{\mathbb{E}\left[u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right) - u(\mathbf{x}(\widehat{w}))\right]}_{\text{substitution effect}}.$$

If the consumption bundles are homogeneous of degree 1, then the gambling incentive does not exist in a competitive consumption market in contrast to the competitive matching market.

5 Efficiency, Inequality and Progressive Tax

In this section, I use an example to illustrate an inherent efficiency-equity trade-off in the matching setting. I present an example in which there exist two equilibria: (1) one efficient equilibrium in which everyone gambles and the inequality in the matching characteristics is maximal, and (2) another inefficient equilibrium in which no one gambles and the inequality in the matching characteristics is minimal. It demonstrates that risk-taking can be socially desirable. It also demonstrates the link between efficiency and inequality. I then propose a remedy, a progressive taxation scheme which eliminates the inefficient equilibrium, encourages the efficient equilibrium, reduces the inequality resulted from equilibrium gambling, and generates a positive tax revenue.

Suppose that mass 1 of men and mass 1 of women are all born homogeneous with characteristic 2. Each can take an investment gamble before participating in the matching market. Each can take either the degenerate investment or the gamble that makes him or her of characteristic 1 with probability of a half or characteristic 3 with probability of a half. The surplus function is $s(x_m, x_w) = x_m x_w$. The sets of characteristics are thus $X_m = X_w = \{1, 2, 3\}$.

Two quite different equilibria arise. In one equilibrium, no one takes a gamble and the matching characteristics are homogeneous; in another equilibrium, everyone takes the gamble and the matching characteristics are heterogeneous.

In the no-gambling equilibrium, everyone enters the matching market as the innate characteristic without gambling so mass 1 of characteristic 2 men and mass 1 of characteristic 2 women are in the matching market. All men and women marry and divide their surplus equally. The equilibrium payoff functions are $v_m^*(1) = v_w^*(1) = 0$, $v_m^*(2) = v_w^*(2) = 2$, and $v_m^*(3) = v_w^*(3) = 4$. Under these equilibrium payoffs, no one can strictly benefit from taking a gamble, as the payoff function $v_m^*(x) = v_w^*(x) = 2(x-1)$ is linear in the characteristics.

In the gambling equilibrium, everyone gambles. The matching market is composed of mass 0.5 of characteristic 3 men, mass 0.5 of characteristic 1 men, mass 0.5 of characteristic 3 women, and mass 0.5 of characteristic 1 women. People of characteristic 3 are matched to each other, and people of characteristic 1 are matched to each other. The equilibrium payoff functions are $v_m^*(1) = v_w^*(1) = 0.5$, $v_m^*(2) = v_w^*(2) = 1.5$, and $v_m^*(3) = v_w^*(3) = 4.5$. Taking the gamble yields an expected payoff of $(0.5 + 4.5)/2 = 2.5$ whereas no gambling yields an expected payoff of 1.5.

The gambling equilibrium is the more efficient equilibrium: the total surplus in the gambling equilibrium is $\frac{1}{2}(3)(3) + \frac{1}{2}(1)(1) = 5$ while the total surplus in the no-gambling equilibrium is only $1(2)(2) = 4$. In fact, the gambling equilibrium is the most efficient a social planner can do given the feasible gambling strategies, and the no-gambling equilibrium is the least efficient a social planner can do.

An undesirable effect of the population-wide risky behavior is the ex-post inequality in their characteristics and payoffs. A remedy to the problem is a progressive tax scheme on the payoffs. As we see in Section 4, the payoff functions are strictly convex. A progressive tax scheme can flatten out the convexity of share payoff function. As long as the after-tax payoffs remain convex, incentives to gamble still exist. Moreover, a carefully designed progressive tax, besides reducing inequality, can also produce positive revenue and eliminate inefficient equilibrium.

Consider the following progressive tax scheme. If a person's payoff is between 0 and 1, then he or she is provided a subsidy so that her after-subsidy payoff is 1. If a person's payoff is between 1 and 3, then he or she is not provided a subsidy, and neither is he or she taxed. If a person's payoff is above 3, then the portion of his or her payoff above 4 is taxed at rate 2/3. The after-tax payoffs in the gambling equilibrium are $v^\tau_m(1) = v^\tau_w(1) = 1$, $v^\tau_m(2) = v^\tau_w(2) = 2$, and $v^\tau_m(3) = v^\tau_w(3) = 3.5$ (the after-tax payoff of characteristic 2 changes because the most he or she can generate now is to match with 1 and gets all the surplus produced). The payoff from gambling is $(1 + 3.5)/2 = 2.25$ and the payoff from no gambling is 2: thus, gambling is still desirable. Furthermore, the net tax revenue is $1 - 0.5 = 0.5$ as all the 3s get taxed 1 and all the 1s get subsidy of 0.5: there is a positive tax revenue. Furthermore, the inefficient no-gambling equilibrium is eliminated. The after-tax payoffs in the no-gambling equilibrium are $v^\tau_m(1) = v^\tau_w(1) = 1$, $v^\tau_m(2) = v^\tau_w(2) = 2$, and $v^\tau_m(3) = v^\tau_w(3) = 3.5$. It is strictly better for people to gamble given these payoffs, as the risks of gambling are partially insured and the gains are large.

A regressive taxation scheme in this example can also eliminate the inefficient no-gambling equilibrium because it increases the convexity of the payoff function, but can only exacerbate the inequality in payoffs in the gambling equilibrium.

A literature exists on the relationship between economic growth and income inequality. Rosen (1997) focuses on the complementarity between income and location. Becker et al. (2005) focus on market for status and generate endogenous income distributions. Ray and Robson (2012) have an equilibrium growth model with endogenous risk-taking. However, none of these papers consider the gambling incentives in a matching market and the tradeoff between efficiency and inequality.

We can also use this example to think about the sunk investments made by workers and firms before they search for their partners in the labor market. The example demonstrates that making risky investments can be not only privately optimal but also socially optimal. Furthermore, when the population is not taking the efficient level of risk and the economy is as a result not reaching the optimal level of growth, a progressive taxation can induce workers and firms to take the efficient risks. Furthermore, the progressive taxation can also reduce the inequality resulted from risk-taking, and generate tax revenue.

6 Conclusion

The competitive structure of the matching market encourages voluntary risk-taking. These gambling behaviors can be profitable even for risk-averse agents taking actuarially unfair lotteries. The fundamental force that drives this type of behavior is competitive matching and bargaining between the two sides of the market, in the Becker-Shapley-Shubik setting, stability and transferability of utilities. This discovery offers some thought on the relationship between efficiency and inequality and helps explain observed risk-taking behavior in the marriage market.

The following two extensions are worth exploring: restricting transferability of utilities and adding search or informational frictions. I have shown the possibility of pre-matching gambles in a setting with perfectly transferable utility and stability. In general, some degree of transferability (imperfect transferability) of utilities is sufficient to drive the sorting and pre-matching gambling, but it is unclear whether the result would hold without any transferability of utilities. If agents have ordinal preferences, and the matching among the agents is stable in the sense of Gale and Shapley (1962), we might also investigate agents' risk-taking behaviors in pre-matching investment in these markets. However, utilities and fair lotteries are not straightforwardly defined with respect to ordinal preferences.

Furthermore, it would be interesting to show that the logic goes through in a model with search frictions. Adding search frictions should not change the basic logic that a man still may strictly benefit from lotteries and the sorting effect is still strong enough for people to take lotteries. Bi-linearity of surplus function may play a crucial role in the extensions. Burdett and Coles (1997) show that when the surplus function is bilinear, there is block matching in equilibrium, that is, agents on both sides are segregated into blocks of matching that can be viewed as classes. The matching technology and bargaining process may have different implications for the incentives on pre-matching investments.

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