An Evolutionary Justification for Non-Bayesian Beliefs and Overconfidence

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People are Overconfident

- Daily observations
  - Reckless drivers, narcissistic Facebook posters, unapologetic exam-takers
- Empirical evidence
  - 88% of American drivers believe that they drive more safely than the median driver (Svenson, 1981)
- Experimental findings
  - 75% of undergraduate participants think they have above median IQ among their peers (Möbius et al., 2012)

People are Positively Biased Bayesians

- *Positive bias*: people tend to over-react to positive signals indicating their possibility of high ability and under-react to negative signals indicating otherwise

- Positive bias especially towards personal characteristics (e.g. IQ, beauty, safe driving)
  - The same undergraduates in the experiment perfectly Bayesian update about others' IQ

Existing Explanations to Positive Bias

- Psychologists, philosophers, anthropologists, economists...
- Cognitive limitations
  - Selective recall and selective information acquisition
- Self-esteem (Benabou and Tirole, 2002)
- Belief utility (Eil and Rao, 2011; Kőszegi, 2006; Möbius et al., 2012)
  - Derive a belief utility from believing to be of high ability

An Evolutionary Justification (Zhang, 2013)

- “Survival of the biased”: Those who have positively biased posteriors (i.e., are overconfident) have more offsprings.
- When the agent is risk-neutral, the evolutionarily optimal posterior is perfect Bayesian.
- When the agent is risk-averse, the evolutionarily optimal posterior is positively biased.
- Most of the people are overconfident if they have concave utility functions.

Model

- Agent A possesses an imperfectly observable trait $x \in \{H, L\}$
  - e.g. IQ, EQ, physical fitness, driving skill
- She has a prior $\mu_0 \equiv \Pr(x = H)$, observes an informative signal $s \in S$, and forms a posterior belief $\mu(s, \mu_0) \equiv \Pr(x = H | s, \mu_0)$
- Signal generating process is known
  - e.g. $\Pr(s | x = H) = p_{Hs}, \Pr(s | x = L) = p_{Ls} \quad \forall s \in S$
  - e.g. perfect Bayesian $\mu^B(s, \mu_0) = \mu_0 p_{Hs} / [\mu_0 p_{Hs} + (1 - \mu_0) p_{Ls}]$

$$\text{logit}\left(\mu^B(s, \mu_0)\right) = \log\left(\frac{\mu_0}{1 - \mu_0}\right) + \log\left(\frac{p_{Hs}}{p_{Ls}}\right)$$

Agent's Problem

With posterior $\mu \equiv \mu(s, \mu_0)$, A chooses an action a to maximize her expected utility,

$$u_A\left(a|\mu\right) = \mu u\left[F\left(a, H\right) - c\left(a\right)\right] + \left(1-\mu\right) u\left[F\left(a, L\right) - c\left(a\right)\right],$$

where

- population growth is $F\left(a, x\right) - c\left(a\right)$: bears offspring $F\left(a, x\right)$ and costs $c(a)$
    - $F(a, x)$ is continuously differentiable, increasing, and concave in a: $F_a > 0$, $F_{aa} \leq 0$, $F\left(0, x\right) = 0$
    - $F(a, x)$ is strictly increasing in x: $F\left(a, H\right) > F\left(a, L\right)$ for all $a > 0$
    - $c\left(a\right)$ increasing and strictly convex: $c'\left(a\right) > 0, c''\left(a\right) > 0$
- Survival utility is $u'\left[F\left(a, x\right) - c\left(a\right)\right] > 0$

Nature's Problem

- Nature N forms perfect Bayesian posterior $\mu^B \equiv \mu^B(s, \mu_0)$ and has the objective to maximize expected population growth

$$u_N\left(a|\mu^B\right) = \mu^B\left[F\left(a, H\right) - c\left(a\right)\right] + \left(1 - \mu^B\right)\left[F\left(a, L\right) - c\left(a\right)\right]$$

Evolutionarily optimal action $a^* = \arg\max u_N(a|\mu^B)$.

- N manipulates A's posterior μ so that A chooses a^* to maximize her expected utility

$$u_A\left(a|\mu\right) = \mu u\left[F\left(a, H\right) - c\left(a\right)\right] + \left(1 - \mu\right) u\left[F\left(a, L\right) - c\left(a\right)\right]$$

Evolutionarily optimal posterior μ^*: $a^* = \arg\max u_A(a|\mu^*)$.

Bias

- Agent's FOC, $\tilde{a}(\mu)$ is the agent's EU-max action given μ,

$$\operatorname{logit}\mu = \log\left|\frac{F_a\left(\tilde{a}(\mu), L\right) - c'\left(\tilde{a}(\mu)\right)}{F_a\left(\tilde{a}(\mu), H\right) - c'\left(\tilde{a}(\mu)\right)}\right| + \log\left[\frac{u'\left[F\left(\tilde{a}(\mu), L\right) - c\left(\tilde{a}(\mu)\right)\right]}{u'\left[F\left(\tilde{a}(\mu), H\right) - c\left(\tilde{a}(\mu)\right)\right]}\right]$$

- Nature's FOC, a^* is the evolutionarily optimal action

$$\operatorname{logit}\left(\mu^B\right) = \log\left|\frac{F_a\left(a^*, L\right) - c'\left(a^*\right)}{F_a\left(a^*, H\right) - c'\left(a^*\right)}\right|$$

- μ^* satisfies $a^* = \tilde{a}(\mu^*)$ if

$$B\left(a^*\right) \equiv \operatorname{logit}\left(\mu^*\right) - \operatorname{logit}(\mu^B) = \log\left[\frac{u'\left[F\left(a^*, L\right) - c\left(a^*\right)\right]}{u'\left[F\left(a^*, H\right) - c\left(a^*\right)\right]}\right]$$

Examples

Example (CRRA)

$u(C) = C^{1-\rho}/\left(1-\rho\right)$, $\rho \geq 1$, $u'(C) = C^{-\rho}$,

$$B\left(a^*\right) = \rho \log \left| \frac{F(a^*, H) - c(a^*)}{F(a^*, L) - c(a^*)} \right|.$$

Example (CARA)

$u(C) = K - \exp(-\alpha C)$, $\alpha \geq 0$, $u'(C) = \alpha \exp(-\alpha C)$,

$$B\left(a^*\right) = \alpha \left[F\left(a^*, H\right) - F\left(a^*, L\right)\right].$$

Evolutionarily Optimal Posterior

Proposition 1

When the agent is risk-averse (risk-neutral/risk-loving), the evolutionarily optimal posterior is positively (not/negatively) biased.

Proof.

$F(\cdot, L) < F(\cdot, H)$, so when $u'' < / = / > 0$,

$$\frac{u'[F(a^*, L) - c(a^*)]}{u'[F(a^*, H) - c(a^*)]} > / = / < 1.$$

and

$$B\left(a^*\right) = \log\left[\frac{u'[F(a^*, L) - c(a^*)]}{u'[F(a^*, H) - c(a^*)]}\right] > / = / < 0$$

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Mutation

- Proportion q true type H agents and proportion $1 - q$ true type L agents.
- Each agent $A \in \mathscr{A}$ observes her parent's true type $x_A \in \{H, L\}$ and inherits the type with probability $1 - \epsilon$ and mutates with probability ϵ.
- After mutation, the true proportions are still $(q, 1 - q)$.
- Now everyone observes an independent signal $s_A \in S$.
- μ_A is the individual posterior belief of being high type after observing individual signal s_A.

Population Posterior

- $\mu_{\mathscr{A}}$ is the collection of individual beliefs $\{\mu_A\}_{A\in\mathscr{A}}$.
- Population posterior: the total expected proportion of people believing they are of high types,

$$q(\mu_{\mathscr{A}}) \equiv \int_{\mathscr{A}} \mu_A.$$

- If everyone uses Bayesian updating, they have Bayesian posterior, and the population posterior is equal to the population proportion,

$$q = q\left(\mu_{\mathscr{A}}^B\right).$$

Inflated Population Posterior

Proposition 2

If most agents are risk-averse in the population, the evolutionarily optimal population posterior belief is strictly greater than the population composition of high type.

Proof.

By Proposition 1, $\mu_A^* > \mu_A^B$ if A is risk-averse. If everyone is risk averse,

$$q(\mu_{\mathscr{A}}^*) = \int_{\mathscr{A}} \mu_A^* > \int_{\mathscr{A}} \mu_A^B = q(\mu^B) = q.$$

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Summary

- The evolutionarily optimal posterior is systematically different from perfect Bayesian posterior.
- If an agent is risk-averse, her evolutionarily optimal posterior belief is positively biased.
- If most agents are risk-averse in the population, the population beliefs are above true population average.

Limitations and Extensions

- It only derives an evolutionarily optimal *posterior belief*, but not an evolutionarily optimal *updating rule*.
- Why and how risk aversion and non-Bayesian belief are evolutionarily optimal at the same time.
  - Why not simultaneous risk neutrality and perfect Bayesian belief?
- Null about *conservative updating*: the magnitude of updating is small for both new positive and negative signals.

References I

Benabou, Roland and Jean Tirole, "Self-Confidence and Personal Motivation," *Quarterly Journal of Economics*, 2002, *117* (3), 871–915.

Eil, David and Justin M. Rao, "The Good News-Bad News Effect: Asymmetric Processing of Objective Information about Yourself," *American Economic Journal: Microeconomics*, 2011, *3*, 114–138.

Kőszegi, Botond, "Ego Utility, Overconfidence, and Task Choice," *Journal of European Economic Association*, 2006, *4* (4), 673–707.

Möbius, Markus M., Muriel Niederle, Paul Niehaus, and Tanya S. Rosenblat, "Managing Self-Confidence: Theory and Experimental Evidence," September 2012. Mimeo.

References II

Svenson, Ola, "Are We Less Risky and More Skillful Than Our Fellow Drivers?," *Acta Psychologica*, 1981, *47*, 143–148.

Zhang, Hanzhe, "Evolutionary Justifications for Non-Bayesian Beliefs," *Economics Letters*, November 2013, *121* (2), 198–201.